CORONAL MASS EJECTIONS: A DIFFERENT MATHEMATICAL APPROACH

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Abstract. We tackle the coronal mass ejection (CME) breakout model via the geometric methods of the theory of dynamical systems. We regard the model stages as phase portraits and discuss them mainly from the mathematical standpoint. The phase-space structure proves to be more and more intricate as time goes by. We emphasize critical points, exotic motions, possibility of chaotic behaviour and escape/capture motion. This paper tries to draw attention to new tools, used in nonlinear analysis (especially in celestial mechanics), as being useful for the solar physics.

Key words: solar physics – coronal mass ejections – dynamical systems.

1. INTRODUCTION

The Sun is a magnetic star. The magnetic field lines move from the solar interior upward in the atmosphere by dynamo action and form magnetic arcades, seat where solar prominences or coronal streamers usually form. These arcades are destabilized due to complex causes and erupt in coronal mass ejections (CMEs). The CMEs are huge magnetic plasma clouds, which leave the Sun and cause serious disturbances in the interplanetary space. After such a magnetic cloud is ejected, the initial magnetic arcade reconnects. The magnetic reconnections could be themselves the releasing factor for a CME. Modelling these phenomena constitutes a challenge for solar physics and, due to the nonlinear developments, most authors use the numerical MHD simulations.

One of the most famous models is that of Antiochos et al. (1999): the breakout model for a CME initiation. Strongly sheared flux forms along the magnetic polarity inversion line. These sheared magnetic lines are opened during the CME. The overlying background magnetic field lines reconnect with the sheared arcade at the null point.

MacNeice et al. (2004; hereafter MN) produced the first numerical simulation of the complete breakout process including the CME initiation, plasmoid formation and ejection, and the magnetic field relaxation after the CME. Their model produces fast

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speed CMEs proving that the breakout model efficiently converts the free magnetic energy into kinetic energy.

The goal of this paper is to present a method complementary to the analytical and numerical ones. We start from the paper MN; our basic idea is to extract more information from the numerical model and to interpret it in terms of plasma motion along the magnetic field lines. So, we interpret the model stages as phase portraits, discuss them from the mathematical standpoint, and formulate some possible developments.

Our idea was already used (Mioc and Dumitrache 2007) for solar prominence models. Now we extend it to CMEs, hoping that such rather unusual methods could enrich the tools of the solar physics. We also hope that our results could model observed situations and structures.

2. METHOD

As in MN, we interpret the motion of the plasma along the magnetic field lines as phase portraits in a phase plane. Using this method, we can obtain extra-information about the plasma motion, and we can suggest the possibility of other scenarios in the observed features.

Following the results of MN (Fig. 1 of their paper), obtained via numerical approach, we have chosen the scenario with four flux systems. All our phase-plane portraits correspond to the projection onto the plane $\phi = 0$ (see MN).

We have to specify that our phase portraits constitute a simplification of the curves obtained numerically by MN. We kept only the general trends illustrated in Fig. 1 of their paper, in order to emphasize all possible phase curves and especially the critical points. To make our plots as simple as possible, we did not label the coordinates (actually, the coordinates are the same as in Fig. 1 of MN). We can say that our phase portraits are to be interpreted as some kind of abstract or generic phase plot.

For our strictly mathematical comments, we do not care about the nature of the field, we consider only the phase-plane structure. By Cauchy's theorem, for every set of initial conditions, a solution exists and is unique. An important point is the timespan on which a solution is defined. Since every solution is related to the lifetime of the CME (finite in both past and future), all solutions encounter singularities.

Another important point is a convention on terminology: just to use the terms of the classical theory of dynamical systems. Following common usage, we shall call the solution curves that start from/end on the solar edge (Fig. 1 in MN and Figs. 1-3 below) *ejection/collision orbits*. Using McGehee-type transformations of the second kind (McGehee 1974), we can regard the collision as a manifold of equilibria for the global flow.

The solution curves that possibly come from/tend to infinity (Fig. 4 below) will be called *capture/escape orbits*. Even not pointed out in Fig. 1 in MN, we assert the existence of singular points, which will be called equilibria: stable (*centers*) or unstable (*saddles*). They generate many interesting orbits (meaning many interesting possible motions) which become more and more complex as time increases.

3. RESULTS

Perusing the drawings of the model of a CME evolution (see Fig. 1 of MN), we identified the essential motions along the magnetic field lines. Reducing them to a simplified phase portrait, we get the following results.

For the initial instant, we have Fig. 1. We remark that most orbits are heteroclinic, ejecting from collision and then ending in collision. There is only one unstable equilibrium (saddle), S_1, which creates heteroclinic orbits that eject from collision and tend to S_1 or conversely. The ejection-collision orbits can be either inside the separatrix defined by S_1, or outside it.

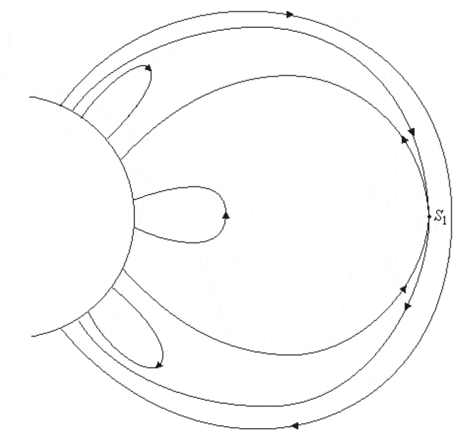

Fig. 1 – The phase portrait corresponding to the initial instant of Fig. 1 of MN.

As time goes by, a second saddle, S_2, appears (Fig. 2). It generates a homoclinic orbit, which shelters a stable equilibrium (center) C_1 surrounded by quasiperiodic and periodic trajectories. Now there are two separatrices generated by the saddles, which generate, in turn, the respective heteroclinic obits. The other phase curves remain the same (all heteroclinic, of ejection-collision type), but of three kinds: inside the separatrix generated by S_2, outside it, but inside the separatrix generated by S_1, and those outside both separatrices.

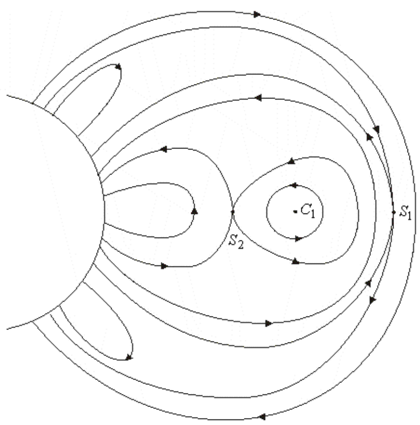

Fig. 2 – The phase portrait at a later time, exhibiting two saddles, a center, a homoclinic orbit, and quasiperiodic and periodic orbits (see also Fig. 1 of MN).

Further on, many more zones of quasiperiodic and periodic orbits appear, some without a center, some with other centers (C_2 and C_3, see Fig. 3), being separated from the first one (Fig. 2) by another homoclinic loop (generated by the saddle S_3), and each other by a double homoclinic loop (generated by the saddle S_4). The other phase curves remain the same (all heteroclinic), but of more kinds, as they lie inside or outside the separatrices. What is important is that the saddles multiply, and this creates a more and more intricate phase portrait.

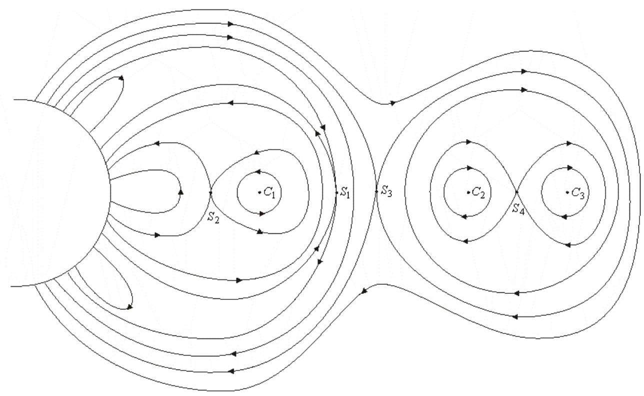

Fig. 3 – The phase portrait at an even later time, exhibiting four saddles, three centers, four homoclinic orbits (from among a double loop), and quasiperiodic and periodic orbits (see also Fig. 1 of MN).

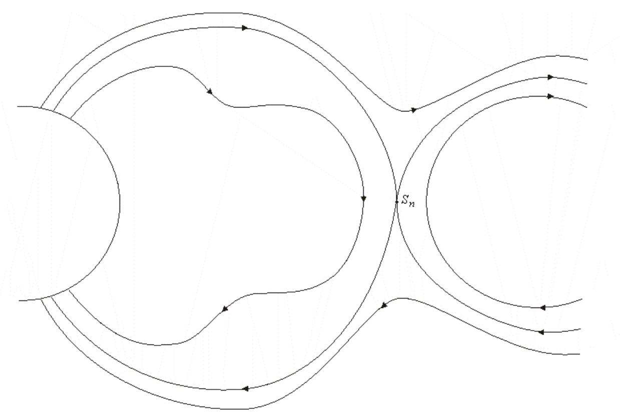

Fig. 4. – Extrapolation of the phase portrait, with possible ejection-escape, capture-collision, and capture-escape orbits.

Now we extrapolate the above results (Fig. 4).

We saw that, as time goes by, the saddles multiply. If, at some instant, a saddle which does not generate a homoclinic orbit (S_n in Fig. 4) appears outside the outer closed curves, then ejection-escape, capture-collision, and capture-escape orbits do appear.

To end, the multiplication of saddles and the existence of homoclinic loops could lead to chaotic behaviour under perturbations.

4. DISCUSSION

The multiplication of the saddles (Fig. 3) could explain the successive magnetic reconnections in the breakout model. Comparing to MN (Fig. 1, last panel), the plasmoid is represented by the phase curves inside the homoclinic orbit generated by S_3. We see the phase curves (and the field lines) that reconnect behind it. Even if the plasmoid is confined by outer closed field lines, these ones could extend very far from the solar surface, such that the plasmoid could reach interplanetary space.

The scenario in Fig. 4, with genuine open field-lines, depends on the supposed appearance of the saddle S_n.

By the powerful tools of the dynamical systems theory, we pointed out a lot of possible motions that can exist in CMEs. The existence of such motions could be confirmed by future observations.

As to a possible chaotic behaviour, its observability is not at hand yet, we only emphasized this possibility.

As a concluding remark, we emphasized a supplementary (and efficient) mathematical tool intended to investigate the CMEs, the most spectacular events in solar activity.

Of course, our approach is only a qualitative (geometric) one, and needs a consistent support of physical interpretation of the phases curves seen, modelled, or proposed. This requires a closer cooperation between solar physicists and nonlinear analysts (especially celestial mechanicians), in order to offer new or (at least) less usual tools to solar features study.

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