D.E.U.S. Multiverse

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Abstract. We interpret the D.E.U.S. (Dimension Embedded in Unified Symmetry) objects embedded in the catenoid of a higher D.E.U.S. object as Stephani Universes. In this assumption, we check for the validity of the Gibbs-Duhem equation, one of the four conditions needed in order that the fluid to evolve in local thermal equilibrium.

Key words: spacetime topology − multiverse - thermodynamic processes.

1. INTRODUCTION

A special class of Stephani Universes (Stephani 1967) can be interpreted as an ideal gas evolving in local thermal equilibrium.

The local thermal equilibrium evolution of a fluid is determined by the following four conditions:

(i) Energy-momentum conservation:

$$\nabla \cdot \mathbf{T} = 0. \tag{1}$$

(ii) The energy density ρ is decomposed in terms of matter density $\Re$ and the specific internal energy ε:

$$\rho = \Re(1 + \varepsilon). \tag{2}$$

(iii) The equation of conservation of matter:

$$\nabla \cdot (\Re\, u) = 0, \tag{3}$$

with u the fluid velocity.

(iv) The thermodynamic quantities temperature T and specific entropy s are related by equations of state compatible with the thermodynamic Gibbs-Duhem relation:

$$T\,ds = d\varepsilon + p\,d(1/\Re). \quad (4)$$

2. D.E.U.S. OBJECTS AS STEPHANI UNIVERSES

In Coll and Ferrando (2005) it is considered the following thermodynamic characterization:

Proposition 1 The necessary and sufficient condition for a non barotropic and non isoenergetic divergent-free perfect energy tensor $\mathbf{T} = (\rho + p)\, u \otimes u - p\, g$ to represent the local thermal equilibrium evolution of an ideal gas is that its indicatrix spacetime function $\chi \equiv \frac{u(p)}{u(\rho)}$ be a non identity function of the variable $\pi \equiv \frac{p}{\rho}$:

$$d\chi \wedge d\pi = 0, \quad (5)$$

with $\chi \neq \pi$.

For the D.E.U.S. object's catenoid we saw that (Popescu 2007), from the external Lorenzian frame, we have:

$$\begin{aligned} u(\rho) &= U^0_{catenoid} \equiv \frac{dt_{FRW}}{dt} = -\frac{1}{r}\frac{\sqrt{\tan^2 t_{FRW} + 1}}{\tan t_{FRW}} \\ u(p) &= U^1_{catenoid} \equiv \frac{dt_{FRW}}{d\phi} = -\frac{1}{r}\sqrt{\tan^2 t_{FRW} + 1} \end{aligned}, \quad (6)$$

cu tan t_{FRW} = $\sqrt{H^2+1} - H$ and H the Hubble parameter. From (6) results:

$$\chi \equiv \frac{u(p)}{u(\rho)} = \tan t_{FRW}\,. \quad (7)$$

Also, because the catenoid is associated to a wave representation (radiation):

$$\pi \equiv \frac{p}{\rho} = 1\,. \quad (8)$$

At tan $t_{FRW} \neq 1$, or $t_{FRW} \neq \pi/4$ (the equality being the signature of the D.E.U.S. object collapse, or, from the FRW external observer's perspective, to the event horizon of the D.E.U.S. black hole), we have $\chi \neq \pi$ and $d\chi \wedge d\pi = 0$. In consequence, inside the catenoid the fluid is in local thermal equilibrium and is described by a perfect energy tensor.

Proposition 2 A non barotropic and non isoenergetic divergence-free perfect fluid tensor verifying (5) represents the local thermal equilibrium evolution of the ideal fluid with specific internal energy ε, temperature T, matter density $\Re$, and specific entropy s given by:

$$\begin{aligned}
&\varepsilon(\rho, p) = \varepsilon(\pi) \equiv e(\pi) - 1 \\
&T(\rho, p) = T(\pi) \equiv \frac{\pi}{k} e(\pi) \\
&\Re(\rho, p) = \frac{\rho}{e(\pi)} \\
&s(\rho, p) = k \ln \frac{f(\pi)}{\rho}
\end{aligned} , \tag{9}$$

$e(\pi)$ and $f(\pi)$ being, respectively,

$$\begin{aligned}
&e(\pi) = e_0 \exp\left\{\int \psi(\pi)\, d\pi\right\} \\
&\psi(\pi) \equiv \frac{\pi}{(\chi(\pi) - \pi)(\pi + 1)} \\
&f(\pi) = f_0 \exp\left\{\int \phi(\pi)\, d\pi\right\} \\
&\phi(\pi) \equiv \frac{1}{\chi(\pi) - \pi}
\end{aligned} . \tag{10}$$

In the case of our D.E.U.S. catenoid, because $\pi = 1$ and $\chi(\pi) = \chi = \tan t_{FRW}$:

$$\psi(\pi)=\frac{1}{2(\tan t_{FRW}-1)}$$
$$\phi(\pi)=\frac{1}{\tan t_{FRW}-1}$$
$$e(\pi)=e_0\exp\{\psi(\pi)\}=e_0\exp\left\{\frac{1}{2(\tan t_{FRW}-1)}\right\}, \tag{11}$$
$$f(\pi)=f_0\exp\{\phi(\pi)\}=f_0\exp\left\{\frac{1}{\tan t_{FRW}-1}\right\}$$

with which:

$$\varepsilon(\rho,p)=e_0\exp\left\{\frac{1}{2(\tan t_{FRW}-1)}\right\}-1, \tag{12}$$

$$T(\rho,p)=\frac{e_0}{k}\exp\left\{\frac{1}{2(\tan t_{FRW}-1)}\right\}, \tag{13}$$

and:

$$s(\rho,p)=k\ln\left\{\frac{f_0}{\rho}\exp\left[\frac{1}{\tan t_{FRW}-1}\right]\right\}=k\ln\left(\frac{f_0}{\rho}\right)+\frac{k}{\tan t_{FRW}-1}, \tag{14}$$

where (Popescu 2007):

$$\rho=p=\frac{15}{256\pi}\frac{m^2}{q^2}\frac{a^4}{r^4}. \tag{15}$$

Results that the specific entropy is:

$$s(\rho,p)=k\ln\left\{f_0\frac{256\pi}{15}\frac{q^2}{m^2}\frac{r^4}{a^4}\right\}+\frac{k}{\tan t_{FRW}-1}. \tag{16}$$

The derivatives of (16) with respect to the radial coordinate r and to the t_{FRW} time of the FRW spacetime are:

$$\begin{aligned} \frac{\partial s}{\partial r} &= 4\frac{k}{r} \\ \frac{\partial s}{\partial t_{FRW}} &= -\frac{k}{1-\sin\left(2t_{FRW}\right)} \end{aligned} . \tag{17}$$

Then:

$$ds = \frac{\partial s}{\partial r} dr + \frac{\partial s}{\partial t_{FRW}} dt_{FRW} = 4\frac{k}{r} dr - \frac{k}{1-\sin\left(2t_{FRW}\right)} dt_{FRW} . \tag{18}$$

Now, in (4) we have (from (12)):

$$d\varepsilon = \frac{\partial \varepsilon}{\partial r} dr + \frac{\partial \varepsilon}{\partial t_{FRW}} dt_{FRW} = -\frac{e_0}{2}\frac{1}{1-\sin\left(2t_{FRW}\right)} \exp\left\{\frac{1}{2\left(\tan t_{FRW}-1\right)}\right\} dt_{FRW} \tag{19}$$

and (from (9), (11) and (15)):

$$d\left(1/\Re\right) = e_0 \frac{256\pi}{15}\frac{q^2}{m^2}\exp\left\{\frac{1}{2\left(\tan t_{FRW}-1\right)}\right\}\frac{r^3}{a^4}\left[4\,dr - \frac{1}{2}\frac{r}{1-\sin\left(2t_{FRW}\right)} dt_{FRW}\right]. \tag{20}$$

In conclusion, the result of (13) multiplied with (18) proves to be equal with $d\varepsilon + p\, d(1/\Re)$, with p from (15). In other words, the Gibbs-Duhem equation (4) is satisfied for the catenoid fluid.

When the catenoid reaches the D.E.U.S. pre-collapse cylinder (or, from the perception of a FRW external observer, at the D.E.U.S. object event horizon) we have $t_{FRW} \to \pi/4$, case in which:

$$s(\rho, p) \to \pm\infty , \tag{21}$$

$$T(\rho, p) = \frac{e_0}{k}\exp\left\{-\frac{1}{2\left(1-\tan t_{FRW}\right)}\right\} \to 0 , \tag{22}$$

representing the "classical" result for the external observer perception of the black hole interior, and also:

$$\Re(\rho, p) = \frac{\rho}{e_0}\exp\left\{\frac{1}{2\left(1-\tan t_{FRW}\right)}\right\} \to \infty , \tag{23}$$

the matter density going to infinity. In the same conditions, the external observer sees that the black hole is having negative specific internal energy:

$$\varepsilon(\rho, p) = e_0 \exp\left\{ -\frac{1}{2(1-\tan t_{FRW})} \right\} - 1 \to -1\,. \qquad (24)$$

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