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COSMIC RAYS ACCELERATION IN WOLF-RAYET STELLAR WINDS

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Abstract. Popescu et al. (2004) gave a model for the observed cosmic rays between $5 \cdot 10^{15}$ and $3 \cdot 10^{18}$ eV. Their source is presumed to be the supernova of stars that explode in their winds. The observed cosmic rays abundance at the source are affected by spallation in the supernova shell, by the difference in ionization degree (being one or two times ionized) at the injection in the supernova shock, the stars with initial masses $15M_{Sun} \leq M \leq 30M_{Sun}$ having a different contribution to them than the stars with $30M_{Sun} \leq M \leq 50M_{Sun}$, this being 2:1 for the elements with $Z \geq 6$. Still, the abundances after these corrections are different by a factor Z_i / Z_{He}, where Z_i is the atomic number for the element *i*. This paper is dedicated to the explanation of this factor and its physical meanings by considering that, prior to the shock injection, the wind particles are radiative accelerated.

Key words: cosmic rays – radiative acceleration – Wolf-Rayet stars.

1. INTRODUCTION

To understand why we need a radiative acceleration of cosmic rays (CRs) and an phase space dispersion before injection in the supernova shock of the particles found in the Wolf-Rayet wind we will do a brief presentation of our previous results (Popescu et al. 2004) on PeV-EeV energy range CRs.

These results show that the ions that can be found in the atmospheres of stars with initial masses of $15M_{Sun} \leq M \leq 50M_{Sun}$ are possible to be the CRs observed particles (whose origin is still an unsettled subject of scientific debate) with abundances affected by spallation and ionization losses. Also it was shown that the mass fraction for one even-*Z* element with $Z \geq 6$ from CRs is $X_i = \alpha X_{i,RSG} + (1-\alpha) X_{i,WR}$.

We have here a different contribution to CRs abundance of stars that explode as supernova in RSG state or in WR state, α factor being approximately equal to 2/3.

Also, ionization loss is responsible for the underabundance in the observed CR elements with FIP $\geq$ 10 eV (the first ionization potential). We can consider Si as reference element for overabundance of elements with FIP ≤ 10 eV (Silberberg and Tsao 1990). The FIP correction factor will be, in this case, 4.0088 ÷ 4.878. In this way, we can see that the elements with FIP less than 10 eV have a mass fraction larger than the elements with FIP greater than 10 eV (and relative to Si), by a factor of ~ 4. This happens to be exactly $Z^2_{injection}$, the initial degree of ionization squared. Therefore, cosmic ray particles of an element with an initial degree of ionization of $Z^2_{injection}$ are more likely to be injected by a factor of 4. The FIP effect is discussed also by Rice et al. (2000, 2001) which suggested that, before the injection at the terminal shock, the pickup ions are preaccelerated via other shocks (e.g., as in the case of the Solar plasma physics, interplanetary shocks). The other proposal is that the particle acceleration takes place directly at the terminal shock by “shock surfing” (Zank et al. 1996).

After introducing also the spallation correction of the mass fractions in the massive stars atmospheres (Tsao et al. 1998) we still remain with a difference between the abundance in massive star winds corrected for all above effects and the observed CRs: $\left(X_i \,/\, X_{He}\right)^{observed CRs} = f \cdot \left(X_i \,/\, X_{He}\right)^{corrected\,wind}$, where

$$f = \left(Z_i \,/\, Z_{He}\right)^{k_i}. \tag{3}$$

The k_i values can be seen in Table 1.

Table 1

Values of k_i for even-Z elements in the case α = 2/3.
The radiative acceleration correction factor is f from (3)

Element *i*	*Z*	k_i
H	1	-
C	6	0.876 ± 0.022
O	8	0.998 ± 0.0265
Ne	10	1.145 ± 0.022
Mg	12	~ 1
Si	14	~ 1
Fe	26	~ 1

Now we can wonder wherefrom this factor is coming. We will see that in a radiative instability (Owocki 1994), thermal instability (Alfvén waves induced – Gonçalves et al. 1998), forward (Lucy 1982) or reverse (Moffat 1994) shocks in the wind, that are driving turbulence, it appears a phase space separation. Also, in a radiative accelerated wind (CAK theory – Castor et al. 1975; MCAK theory – Pauldrach et al. 1986; etc.), the ions are differently accelerated in function of the resonant line absorbtions of photons.

2. WOLF-RAYET (WR) CLUMPED WIND MODELS

Spherically-symmetric homogeneous wind models predict IR-fluxes smaller than the observed ones, and, in many cases, smaller than the observed fluxes of HeI lines if matter density is calibrated through the observed radio-fluxes (Nugis 1994). These discrepancies can be removed by clumped wind models.

For WR stars there exists also a serious momentum problem, the mass-loss rates for these stars, determined from radio-fluxes, exceed in some cases 30–50 times the single scattering limit $L / c / v_{\infty}$ (Nugis 1994). The clumpy wind could lead to a decrease in the estimate of the mass-loss rates by a factor ≥ 3 compared to homogeneous winds of the same free-free or subordinate line flux (Moffat 1994).

Observations have proven that density inhomogeneities exist in the wind of WR stars (Moffat et al. 1988). The spectra of WR stars are dominated by broad emission lines of helium, with lines of nitrogen in stars of the WN sequence, or carbon and oxygen in stars of the WC/WO sequence. Lépine et al. (1999) studied the line-profile variations (LPVs), by high-resolution spectroscopy, in the HeII λ5411 emission line of four WR stars of the WN sequence (HD 96548: WR 40, HD 191765: WR 134, HD 192163: WR 136, HD 193077: WR 138) and in the CIII λ5696 emission line of five WR stars of the WC sequence (HD 164270: WR 103, HD 165763: WR 111, HD 192103: WR 135, HD 192641: WR 137, HD 193793: WR 140). The LPVs present systematic patterns: they all consist of a number of relatively narrow emission subpeaks that tend to move from the line centers toward the line edges. Lépine et al. (1999) introduced a phenomenological model that depicts WR winds as being made up of a large number of randomly distributed, radially propagating, discrete wind emission elements (DWEEs). This model was used to simulate LPVs patterns in emission lines from a clumped wind. They analyzed the general properties of the LPV patterns with the help of the multiscale, wavelet method. Lépine et al. (1999) investigated the effects on the LPVs of the local velocity gradients, optical depths, various numbers of discrete wind elements, a statistical distribution in the line flux from individual elements, and, also, how the LPV patterns are affected by the velocity structure of the wind and by the extension of the line-emission region (LER). They found that a large number ($\geq 10^4$) of DWEEs must be used to account for the LPV, that the mean duration of subpeak events, interpreted as the crossing time of DWEEs through the LER, is consistent with a relatively thin LER. As a consequence, the large emission-line broadening cannot be accounted for the systematic radial velocity gradient from the accelerating wind. Rather, emission-line broadening must be dominated by the large "turbulent" velocity dispersion suggested by the LPV patterns.

Grosdidier et al. (1997) observed the clumped structure of the ejection-type M1-67 nebula that surounds the WR 124 star (see the image in Hα of M1-67 taken with HST-WFPC2; Grosdidier et al. (1998). The clumped structure of the ejections of WR 124 stellar object is visible in the image). The kinematics of the same nebula was studied with the aid of a scanning Fabry-Perot interferometer in combination with a two-dimensional detector at Université Laval. In this way it was obtained a high resolution velocity field map (see the smoothed velocity map for heliocentric radial velocities of the "red" component of M1-67 taken with the Fabry-Perot interferometer of the Canada-France-Hawaii Telescope MOS/SIS (Grosdidier et al. 1998). The phase space velocity dispersion in the ejections of WR 124 stellar object is visible in the image); (Grosdidier et al. 1997). On the HST-WFPC2 image of M1-67 was done a structure function analysis, with the help of two-dimensional wavelets (to isolate stochastic structures of different characteristic size), being identified the self-similarity (fractality) in the fields of increments.

2.1. THE STRUCTURE FUNCTION ANALYSIS

To analyze the phase-space properties of discrete wind emission elements from nebulae around massive stars, the wavelet method can be used to perform the structure function analysis (Muzy et al. 1993). The image will be considered as a 2-dimensional function *f*. The field of increments of f over a projected spatial distance r at the pixel (x, y): $\Delta f[r;(x, y)] = f[(x, y) + \mathbf{R}/2] - f[(x, y) - \mathbf{R}/2]$, by the continuous wavelet transform, $T_\Psi(f)$, is $\Delta f[r;(x, y)] \approx T_\Psi(f)[r;(x, y)] = r^{-2} \times (f \otimes \Psi_r)[r;(x, y)]$, where $\mathbf{R}$ is a two-dimensional increment vector of length *r*, $\otimes$ denotes a convolution, and Ψ_r is the analyzing wavelet at the spatial scale *r* (Muzy et al. 1993). The so-called mother-wavelet, Ψ, is the two-dimensional "Mexican hat" (the second derivative of $\exp[-(x^2 + y^2)/2]$ and $\Psi_r(x, y) = \Psi(x/r, y/r)$; Farge 1992). A mother-wavelet function must be such that it has zero mean: $\int_{-\infty}^{\infty} \Psi(r)\,\mathrm{d}r = 0$. Furthermore, it must be localized in space (Lépine and Moffat 1999):

$$\int_{-\infty}^{\infty} [\Psi(r)]^2\,\mathrm{d}r = C\,. \tag{7}$$

Because the Mexican hat wavelet is orthogonal to polynomials of orders 0 and 1, the wavelet transform is insensitive to linear trends, hence linear nonstationary components, in the signal (or image).

The statistical moments of $|\Delta f[r;(x, y)]|^p$ (the two-point correlation function known as "structure function of order *p*") can be estimated through those of $|T_\Psi(f)[r,(x, y)]|^p$, by averaging over $\left\langle |\Delta f(r)|^p \right\rangle \approx C \iint |T_\Psi(f)[r;(x, y)]|^p\,\mathrm{d}x\mathrm{d}y$, where *C* is the constant that can be determined from (7). We can use scaling laws of the form (Grosdidier et al. 2001):

$$\left\langle \left| \Delta f(r) \right|^p \right\rangle \approx r^{\zeta(p)} . \tag{9}$$

By definition, $\zeta(0) = 0$ and $\zeta(p)$ is a smooth, differentiable, monotonically nondecreasing, concave function of p (if the signal has absolute bounds), no matter how rough the data are. Continuous signals satisfy $\left\langle \left| \Delta f(r) \right|^p \right\rangle \approx r$, hence $\zeta(p) \propto p$. Processes with $\zeta(p) \propto p$ are called "monoaffine" (affine) or "monofractal", whereas processes with variable $\zeta(p)/p$ are called "non-affine" (in particular "multiaffine" or "multifractal").

Doing the structure function analysis of the velocity field on M1-67, Grosdidier et al. found $\zeta(1) \approx 0.48$–0.49 and $\zeta(2) \approx 0.90$–0.91.

In the context of the "universal multifractals" (Schertzer and Lovejoy 1987) the multifractals are the result of multiplicative cascades. A continuous-scale limit of such processes lead to the family of log-infinitely divisible distributions that have a Gaussian or Lévy stable generator. For universal multifractals we have:

$$\begin{gathered} \zeta(p) = p\zeta(1) + (C_1 / |\alpha - 1|)(p^\alpha - p), \quad \alpha \neq 1; \\ \zeta(p) = p\zeta(1) - C_1 \ln p, \quad \alpha = 1, \end{gathered} \tag{10}$$

where $C_1 \leq 2$ is an intermittency parameter and $0 < \alpha < 2$ is the Lévy tail index.

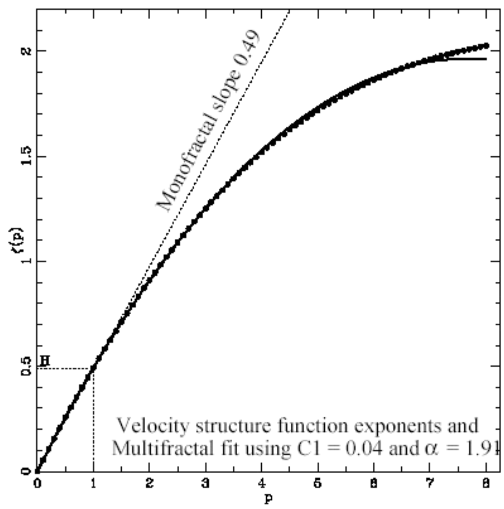

Fig.1 – Velocity structure function analysis of M1-67. The corresponding $\zeta(p)$ function (dots) proves the multiaffinity of the velocity field in the nebula. The solid curve is a universal multifractal fit (Grosdidier et al. 2001).

For testing the universal multifractal behavior it can be used the "double trace moment" (DTM) method (Lavallée et al. 1991). Through DTM, Grosdidier et al. were able to estimate the value $\alpha \approx 1.91$ and, after that, through least-squares method, the value of $C_1 \approx 0.04 \pm 0.01$ for the particular case of M1-67 nebula (see Fig. 1).

2.2. TURBULENCE

Now, knowing the values of $\zeta(p)$ for different values of p, we have the scaling behavior (9) for $r^{\zeta(p)}$.

Turbulence is a form of dissipation in which a cascading process normally transfers energy from the larger to smaller eddy scales (or sometimes the reverse). Depending on the physical constraints during the transfer, different scaling laws may result (Moffat 1994).

The Navier-Stokes equations are invariant under the rescaling $x \to x\lambda^{-1}$, $\nu \to \nu\lambda^{-H}$, and $t \to t\lambda^{H-1}$ (Sylvestre et al. 1999). For example, assuming that ε, the energy flux to smaller scales, is a scale-invariant quantity, it is found that $H = 1/3$, and dimensional analysis leads to the scaling law $E(k) \propto k^{-5/3}$ for the energy density in the momentum space. In terms of Elssaser variables, eddies fall into two classes depending on their direction of propagation along the magnetic field lines: interactions between eddies belonging to different classes are less likely, thus weakening the energy transfer (Iroshnikov 1963; Kraichnan 1965). Consequently, the characteristic interaction time $(\tau_{eddy} / \tau_A)^a$, where τ_A represents the characteristic time for Alfvén waves, and a is some positive constant (Politano and Pouquet 1995). The scaling relation becomes: $E(k) \propto k^{-1-2/(a+3)}$, with $a = 1$ corresponding to Iroshnikov-Kraichnan theory.

It is found experimentally that in hydrodynamic media the Kolmogorov scaling law is generally not respected for individual realizations, and even in estimates of ensemble average the exponent differs from 5/3. The discrepancy can be attributed to intermittency (fluctuations in ε due to small-scale non-linear structures).

From the scaling behavior via the structure function analysis we can obtain the power spectrum estimation as a function of the modulus of the wave vector k. Explicitly, $E(k) \propto k^{-(\beta+1)}$, where β is the scaling exponent, or the spectral slope.

For Kolmogorov scaling, $H = 1/3$, $\beta + 1 = 5/3$. The invariant at the rescaling power, H, for $H = \zeta(q)/q$ with q a specified p value in (10), has the expression $H = \beta/2 + C_1(q^{\alpha-1} - 1)/|\alpha - 1|$. When we have a continuous signal ($\langle|\Delta f(r)|\rangle \propto r$) with, e.g., a Kolmogorov scaling law $H = 1/3$, it results that $H = \zeta(1)/1 = \zeta(1)$, and, knowing that $\beta + 1 = 5/3$, we obtain

$$H = \zeta(1) = \beta / 2 . \tag{14}$$

If $q = 2$ and $\alpha > 1$ (Sylvestre et al. 1999):

$$H = \beta / 2 + C_1 (2^{\alpha-1} - 1) / (\alpha - 1) . \tag{15}$$

For $q = 1$ and $\zeta(1) \approx 0.48$–0.49 (Grosdidier et al. 2001) and $H = \zeta(1)/1$, it results from (14) that $H \sim 1/2$ and $E(k) \sim k^{-2}$. For $q = 2$ (in (15)) and $\zeta(2) \approx 0.9$–0.91 (Grosdidier et al. 2001), $H = \zeta(2)/2$ and at $\zeta(1) \approx 0.48$–0.49, it results that $E(k) \sim k^{-1.9}$, suggesting that the dynamics is essentially shock dominated (for which $E(k) \sim k^{-2}$). The dynamics on the case of M1-67 nebula is possible to be shock dominated ("saw tooth" turbulence) taking in account that WR 124 is a component in a binary system (HD 156327) (van der Hucht 2001), where the companion of the WR star is a B0III-I spectral type star.

The significance of each of the three universality parameters on the multifractal field can be described as follows:

- C_1 corresponds to the codimension of the mean field, and thus distinguishes between a field whose mean is dominated by a few localized peaks (large C_1), and one with a mean dominated by a larger proportion of its surface (small C_1; for non-fractal such as white noise, $C_1 = 0$);
- H is a measure of the degree of non-conservation of the field, or qualitatively a measure of its smoothness (large values of H corresponding to smoother fields). For example, in usual hydrodynamic turbulence the energy flux to smaller scales is conserved ($H = 0$), whereas the velocity shears have the Kolmogorov value $H = 1/3$;
- α is the degree of multifractality (a measure of the deviation from the monofractal case). As α is the Lévy index of the multifractal generator, we have the restriction $0 \le \alpha \le 2$, with $\alpha = 0$ and $\alpha = 2$ corresponding to monofractal (β model) and log-normal models, respectively.

Averaging the universal parameters over seven bipolar nebulae (Sylvestre et al. 1999) - GGD 18, V380 Orionis, LkHα 101/NGC 1579, LkHα 233, PV Cephei, V645 Cygni (GL 2789), and V633 Cassiopeiae (LkHα 198), Sylvestre et al. obtained: $\alpha = 1.96 \pm 0.02$; $C_1 = 0.04 \pm 0.02$; $H = 0.7 \pm 0.2$. At this H value corresponds a power spectrum $E(k) \propto k^{-3/2}$, indicating that the turbulence present in those bipolar nebulae is compressible. The result of wavelet analysis on the cloudlets from GMC L1551 gave a fractal dimension $D = 2.35 \pm 0.01$ (Moffat et al. 1994a), comparable with the one in the WR 135 (a WR star without companion and of WC9d spectral type) nebula, $D = 2.32 \pm 0.07$ (Moffat 1994). This is actually the fractal dimension that results from wavelet and structure function analysis on all compressible turbulence-dominated phenomena.

The observed scaling and mass-spectrum laws strongly imply that, in general, in the WR star without companion case, we have anisotropic, supersonic, compressible turbulence, which increases outwards in the wind. Still, the re-arrangement of single scattering electrons into clumps cannot explain the observed polarization variability, unless the clumping occurs deep in the wind, close to its point of origin (Moffat 1994).

2.3. SELF-SIMILAR STRUCTURES

Until now we made estimations for the turbulence type, for the multifractality, and we saw that we must have "clumps" ("blobs") in the wind of WR stars. What we cannot tell yet is how extended are these clumps for different distances from the base of the wind. We can find a solution to this problem by supposing that what we see is the result of one (or more) self-similar (monoaffine) structure(s) embedded in a random component.

We can consider that the particles belong to two different distributions g_1 and g_2, occupying the same sample volume *V*, each one with its own correlation function referred to that volume. We take g_1 to be the observer-homogeneous self-similar distribution and g_2 a random distribution. If we let N_e denote the number of objects (electrons) belonging to a cluster (blob), the number of blobs in the sample is then given by $n_b = N_{e1} / N_{e2}$ and the fraction f_e of electrons belonging to blobs is $f_e = N_{e1} /(N_{e1} + N_{e2})$, where N_{e1} and N_{e2} are the total number of electrons in the two systems considered. If we further assume that the component g_2 is distributed randomly, then $\xi_{e1e2} = \xi_{e2e2} = 0$, and $\xi_{e1e1} = \xi_{bb}$. So, the correlation function is $\xi_{bb} = (1/ f_e^2)\xi_{ee}$ (Calzetti et al. 1988).

The addition of a random component to a pure fractal structure is therefore equivalent to multiplying the total system correlation function by $f_e^2 < 1$. This fact will have an effect on the general form of the two-point spatial correlation function: $\xi_i(r) = A_i r^{-\gamma} + q_i(r)$, where $\gamma = 3 - D$ (3 is the space dimensionality and *D* the fractal dimension), and the function $q_i(r)$ should: have a negligible effect for the distances *r* for which $A_i r^{-\gamma} > 1$; satisfy the identity: $\int_0^{R_S} A_i r^{2-\gamma}\mathrm{d}r = -\int_0^{R_S} q_i(r) r^2 \mathrm{d}r$, which comes from the normalization condition $\int_0^{R_S} \xi(r) r^2 \mathrm{d}r$ (Calzetti et al. 1989); satisfy $q_i(r) \geq -1$ for large *r* because the probability of finding a particle in the volume d*V* at the distance *r* from any given particle of the system must be nonnegative. In the above equations R_S is the radius of the sample.

Then, the two-point spatial correlation function becomes: $\xi_{ee} = A_{ee} r^{-\gamma} + \mu$, with $A_{ee} = f_e^2 A_{bb}$, $-1 < \mu < 1$, and $\mu = -f_e^2$. When $f_e < 1$, the differential density referred to electrons becomes (Calzetti et al. 1988):

$$n_{de}(r) = \langle n_e \rangle [f_e^2 A_{bb} r^{-\gamma} + 1 - f_e^2]. \tag{21}$$

Because we have two different physical systems superimposed, $n_{de}(r)$ is not the actual differential density distribution seen by any observer belonging to the set, but should instead be interpreted as an average over all the density distributions seen by all the observers belonging to the system. It is important to emphasize that we have $A_{bb} \neq A_{ee}$, where $A_{bb} = (1 - \gamma / 3) R_S^{\gamma}$ (Calzetti et al. 1989).

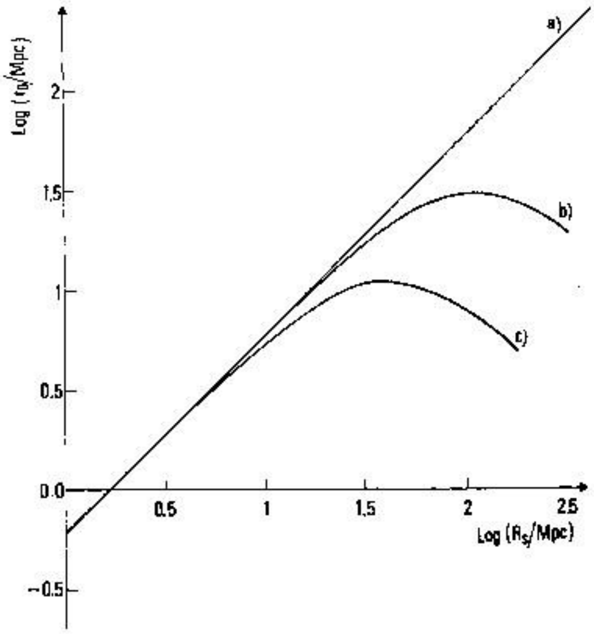

Fig.2 – The dependence of the correlation length r_0 on the sample radius R_S is given here for a pure fractal structure ($K'/K = 0$, line (a)) and for a self-similar structure embedded in a random component ($K'/K \neq 0$) for some selected values of K'/K. For any given value of $K'/K \neq 0$ and for large enough sample radii R_S the dependence of r_0 on R_S is far from linear. Line (b) refers to a value of $K'/K = 2.16 \times 10^{-4}$ and line (c) to $K'/K = 1.37 \times 10^{-3}$ (Calzetti et al. 1988).

While A_{bb} is simply a function of the sample radius and of the fractal dimension, A_{ee} have to take into account the presence of the factor f_e^2.

Since N_{e1} and N_{e2} have clearly different dependencies on the sample radius

$R_S : N_{e1} = KR_S^{3-\gamma}$, $N_{e2} = K' R_S^3$ (Calzetti et al. 1988), where K and K' are constants of proportionality, f_e itself will be a function of the sample radius R_S and γ, and we have

$$f_e = KR_S^{3-\gamma} /(KR_S^{3-\gamma} + K' R_S^3) = KR_S^{-\gamma} /(KR_S^{-\gamma} + K'). \tag{24}$$

As consequence, the R_S-dependence of A_{ee} will be not a simple power law, but:

$$A_{ee} = (1-\gamma/3)K^2 R_S^{-\gamma} /(KR_S^{-\gamma} + K')^2 . \tag{25}$$

We can define the quantity $r_0 \equiv (A_{ee})^{1/\gamma}$ that is the correlation length of the sample size (Einasto et al. 1986). The behavior of r_0 as function of R_S and of the value of the ratio K'/K is shown in Fig. 2.

If we will be able to find a physical correlation between the structure function of order p for a multiaffine situation and the correlation function and length for a self-similar structure embedded in a random component, we will be able to find the values taken by K'/K for particular cases of WR nebulas (for example, for M1-67).

3. NUMERICAL MODEL

3.1. THE RADIATIVE ACCELERATION

A different acceleration for the atomic species that can be found in the winds of WR stars (considered as having the same ionization degree) will give phase space dispersion. This means that the acceleration as function of the local bulk velocity for oxygen must be bigger that the one for carbon, and the carbon acceleration bigger that the one for helium. The above affirmation is sustained by the radiation-driven wind model (Cassinelli 1979) condition in which momentum is transferred from the radiation field to the gas by scattering of radiation in spectral lines. The radiative acceleration for a particular element found in the wind is given as the sum of all the radiative accelerations provided by single lines (Castor et al. 1975). In CAK (after Castor, Abbott and Klein) theory it is shown that this sum can be parameterized by

$$g_{rad} = [cN(r)]^{-1}\sigma_{th}(r)\,\sigma_B T_{eff}^4 M_{CAK}(t), \tag{26}$$

where $M_{CAK}(t) = kt^{-\alpha}$ is the force multiplier and encapsulates the atomic physics of the line list for numerical computation, c –speed of light, $N(r)$ – particle density at the distance r from the base of the wind, σ_B – Stefan-Boltzmann constant, T_{eff} – the effective temperature at the photospheric radius R_*. In our work the effective temperatures and also the terminal velocities v_∞ and the stellar masses for Galactic WR stars were taken from García-Segura et al. (1996), from Crowther et al. (1995), and from van der Hucht (2001) (from where we are making also a selection for WR stars without companion after their spectral type). In the expression of the force multiplier the constants k and α represent the number of scattering lines and the ratio of weak to strong lines, respectively (Abbott 1980) and, for computational purposes, were taken from Pauldrach et al. (1986). The depth parameter t is defined by (v_{th} is the thermal velocity of the carbon ion):

$$t = \sigma_{Th} v_{th} (\mathrm{d}v / \mathrm{d}r)^{-1} . \tag{27}$$

The Thompson total cross section for scattering of radiation has the expression

$$\sigma_{Th} = \frac{8\pi}{3}\left[e^2 / \left(m_e c^2\right)\right]^2 = 6.65 \cdot 10^{-25} [cm^2] . \tag{28}$$

The radiative acceleration in the equation (26) is written in the isothermal case. A more accurate radiative acceleration takes into account a temperature distribution for a spherically grey atmosphere in radiative equilibrium (Milne-Eddington temperature distribution) (Lucy et al. 1993):

$$T^4(r) = \frac{1}{2} T_{eff}^4 \left(2W + \frac{3}{2}\tilde{\tau}\right), \tag{29}$$

where:

$$W = \frac{1}{2}\left[1 - \sqrt{1 - \left(\frac{R_*}{r}\right)^2}\right], \tag{30}$$

and the optical depth:

$$\tilde{\tau} = \int_r^\infty \sigma_{eff}\, \rho(r) (R_* / r)^2\, dr . \tag{31}$$

The reference radius is chosen so that:

$$\tilde{\tau}(r = R_*) = 2/3 . \tag{32}$$

When the stellar luminosity L is not r dependent, the scattering coefficient σ_{eff} will be:

$$\sigma_{eff} = \frac{4\pi GMc}{L}\left[1 + 2\xi\left(-w\frac{dw}{dx}\right)\right]. \tag{33}$$

In (33) G is the gravitational constant, M, the stellar mass, $w = v/v_\infty$, $x = R_c/r$ (R_c is the core radius within which the star is essentially in hydrostatic equilibrium), $\xi = (v_\infty / v_{esc})^2$ (at $r = R_c$ the velocity is equal with the escape velocity, v_{esc}).

The empirical formula which was used in CAK theory (Castor et al. 1975) for correlating v_∞ and v_{esc} is:

$$v_{esc} = \sqrt{(1-\alpha)/\alpha} \cdot v_\infty . \tag{34}$$

In our model we don't find useful to take a $T(r)$ distribution, the radiative acceleration from (26) being taken in the isothermal approximation. This is because the particles are beginning to be injected in the supernova shock very close to the stellar surface, and we are interested how those particles behave, before injection, till one or two stellar radii distance in the atmosphere, and if there appears a phase space separation as function of their type. Because is essential what happens close to the photosphere, we can't work in the radial streaming approximation for the photons that drive the wind (Abbott 1980), by using instead of (27) the "exact" depth parameter (Pauldrach et al. 1986):

$$t_{ex} = \frac{t(dv/dr)}{(1-\delta^2)(v/r) + \delta^2(dv/dr)}, \tag{35}$$

which leads to the improved force multiplier:

$$\overline{M} = \frac{2}{1-\delta_*^2}\int_{\delta*}^{1} M_{CAK}(t_{ex})\delta \, d\delta , \tag{36}$$

(where δ is the cosine of the angle between the photon direction and the outward normal on the spherical surface element, and $\delta_* = \sqrt{1-(R_*/r)^2}$) or explicitly gives:

$$\overline{M}(t, v, dv/dr, r) = M_{CAK}(t)\frac{2}{1-\delta_*^2}\int_{\delta*}^{1}\left[\frac{(1-\delta^2)(v/r) + \delta^2(dv/dr)}{(dv/dr)}\right]^{\alpha} \delta \, d\delta . \tag{37}$$

3.2 *VELOCITY AND ACCELERATION LAWS*

In the previous section we said that in order to have a phase space dispersion we must first have: $a_O(v_O) > a_C(v_C) > a_{He}(v_{He})$.

The most popular velocity law (so-called β-law – Castor et al. 1979) which describes the bulk motion of the accelerated bulk material is:

$$v(r) = v_\infty (1 - R_* / r)^\beta . \quad (38)$$

In an approximation of point source stars, the CAK theory predicts a velocity law with β = 0.5. When are considered also the finite disk effects, the CAK theory yields β = 0.8 (Friend et al. 1986). Puls (1996) successfully used a value of β = 1 for the prediction of OB stars mass loss. Still, observations of LPV subpeaks in WR winds, suggested much larger values of β (Robert 1994), the spectral analysis of WR spectra with a clumped wind model (Schmutz 1997) being consistent with these values of β ≅ 4-8.

The acceleration law that follows from (38) is:

$$a(r) \equiv dv / dt = v(dv / dr) = \frac{v_\infty^2}{R_*} \left(\frac{R_*}{r} \right)^2 \beta \left(1 - \frac{R_*}{r} \right)^{2\beta - 1} , \quad (39)$$

or, as function of the local bulk wind velocity:

$$a(v) = \beta \frac{v^2}{R_*} \left[\left(\frac{v}{v_\infty} \right)^{-1/2\beta} - \left(\frac{v}{v_\infty} \right)^{1/2\beta} \right]^2 . \quad (40)$$

In Fig. 3 is illustrated the $a(r)$ and $\beta a(v)$ behavior for different values of the parameter β (Lépine et al. 1999). Apart from other important remarks that can be made related to these representations, it can be seen that, for the same particle velocities we have a spatial separation (into blobs?) and a constant $\beta a(v)$. For our purposes it is also useful to see that, for a star with a specified photospheric radius R_*, at a constant r and different β values we have a phase space dispersion due to different particle velocities.

If the blobs follow a velocity law (38) the time spent between $x_0 = r_0 / R_*$ and $x = r / R_*$ is:

$$T = \frac{R_*}{v_\infty} \int_{x_0}^{x} dx \left(1 - \frac{1}{x} \right)^{-\beta} , \quad (41)$$

so that most time is spent near x_0 for likely β values. For $x_0 = 1$, T is infinite (unless β<1) for all x. Statistically speaking, this means that for the (38), β-law blobs should never be observed other than at $x = 1$. In addition, any $v(r)$ law with $v = 0$ at $x = 1$ requires infinite wind density for mass flow continuity (Brown et al. 1995). This difficulty can be avoided by allowance for an initial velocity $v(x = 1) = v_{th} = \varepsilon v_\infty$, where

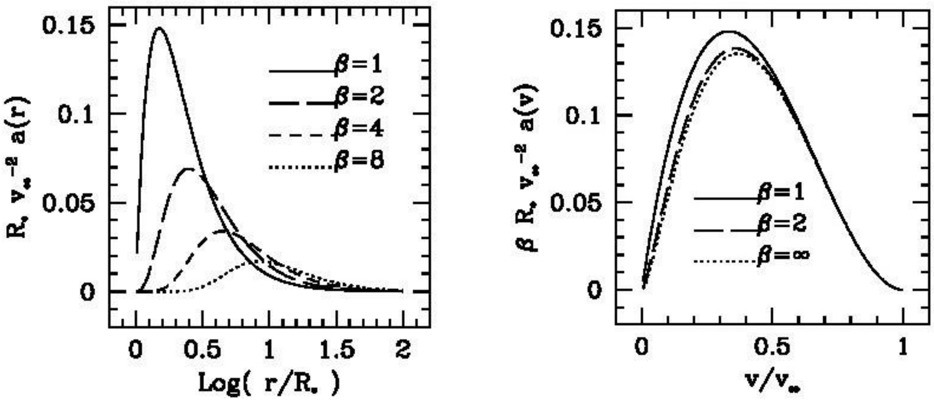

Fig. 3 – Illustration of the β-law. *Left panel*: Dimensionless acceleration plotted as function of radius; *Right panel*: The β-law in velocity space (it can be seen the same general shape for the β values). (Lépine et al. 1999)

v_{th} is the thermal velocity of the carbon ion near the "dynamical photosphere". Then, the modified β-law becomes:

$$v(r) = v_\infty \left[\varepsilon + (1-\varepsilon)(1 - R_* / r)^\beta \right]. \tag{42}$$

At the same ε values the acceleration in the first two stellar atmosphere radius distances is more efficient for small β values. The conclusions that follows from Fig. 3 and Fig. 4 is that we can have a phase space separation for different atomic species that are present in the WR wind and, if we want to have $a_O(r_O) > a_C(r_C) > a_{He}(r_{He})$, we must have first $\beta_O < \beta_C < \beta_{He}$, which means that (see Fig. 3) we need that $a_O(v_O) > a_C(v_C) > a_{He}(v_{He})$ at the same velocity value.

3.3 *NUMERICAL MODEL EQUATIONS*

For a phase space separation we take the factor *f* from the equation (3) equal with the ratio of accelerations (see eq. (40)):

$$\frac{a_C(v_C)}{a_{He}(v_{He})} = \frac{\beta_C}{\beta_{He}}\left(\frac{v_C}{v_{He}}\right)^2 \left[\frac{\left(\frac{v_C}{v_\infty}\right)^{-\frac{1}{2}\beta_C} - \left(\frac{v_C}{v_\infty}\right)^{\frac{1}{2}\beta_C}}{\left(\frac{v_{He}}{v_\infty}\right)^{-\frac{1}{2}\beta_{He}} - \left(\frac{v_{He}}{v_\infty}\right)^{\frac{1}{2}\beta_{He}}}\right] = \left(\frac{Z_C}{Z_{He}}\right)^{k_C} \cong 3^{k_C}, \quad (43)$$

$$\frac{a_O(v_O)}{a_{He}(v_{He})} = \frac{\beta_O}{\beta_{He}}\left(\frac{v_O}{v_{He}}\right)^2 \left[\frac{\left(\frac{v_O}{v_\infty}\right)^{-\frac{1}{2}\beta_O} - \left(\frac{v_O}{v_\infty}\right)^{\frac{1}{2}\beta_O}}{\left(\frac{v_{He}}{v_\infty}\right)^{-\frac{1}{2}\beta_{He}} - \left(\frac{v_{He}}{v_\infty}\right)^{\frac{1}{2}\beta_{He}}}\right] = \left(\frac{Z_O}{Z_{He}}\right)^{k_O} \cong 4^{k_O}, \quad (44)$$

where $k_C = 0.764 \pm 0.022$ and $k_O = 0.984 \pm 0.0265$ (see table 1) and the He velocity is a (42) β-law:

$$v_{He}(r_{He}) = v_\infty \left[\varepsilon + (1-\varepsilon)(1 - R_* / r_{He})^{\beta_{He}}\right]. \quad (45)$$

In (43), (44), and (45), β will take values between 4 and 8 (Schmutz 1997) (with the specification that $\beta_O < \beta_C < \beta_{He}$. The photospheric radius will be determined from the relation:

$$R_* = \sqrt{L_{nuc} / (4\pi\sigma_B T_{eff})}, \quad (46)$$

with T_{eff} the effective photospheric temperature, σ_B - the Stefan-Boltzmann constant, and L_{nuc} - the nuclear luminosity of the star, given by an empirical form of the mass-luminosity relation (Langer 1989a):

$$\log(L_{nuc} / L_\Theta) = a_0 + a_1 \log(M / M_\Theta) + a_2 [\log(M / M_\Theta)]^2, \quad (47)$$

where the fitting coefficients found by Heger et al. (1996) for L_{nuc} are a_0 = 2.507301, a_1 = 3.237961, a_2 = -0.610150, and a standard deviation of 0.003188.
(43), (44), and (45) will be the first three equations from a sistem of seven with seven variables. We will try to prove that at any given r, $30M_\odot \le M \le 50M_\odot$, and ε (from the non-zero initial velocity condition in (42)) we are having phase space dispersion, meaning a convergence for the system of equations. The variables of the sistem are:
- The distances at which the particles reach in the wind before their injection in the supernova shock. Assuming that C, O and He (the choice of these atomic species will

be later explained) are becoming wind particles at the same moment of time and that are simultaneously injected in the supernova shock. The last approximation is possible because the bulk velocity in the wind is in the best case 0.1 from the shock velocity. So, the quantities that are to be determined are $(\Delta r)_1 \equiv a = r_C - r_{He}$ and $(\Delta r)_2 \equiv b = r_O - r_{He}$ (in (45) will be made the substitution $r_{He} = r_C - a$), where $r_C \equiv r$ will be the distance at which is reaching the carbon in the wind and it will be given (through an iteration loop) values from 1 R_* to 3 R_* with the step smaller than the Alfvénic damping lengths. The Alfvénic damping lengths can be understood as limits to the size of the formed blobs (self-similar domains) through thermal instability due to Alfvén waves (Gonçalves et al. 1998). Gonçalves et al. found that the blob diameters must be $3\times 10^5 \le d_{blob} \le 1.4\times 10^6$ cm. Consequently, our correlation length (r_0) and our iteration step ($r_C \to r_C + \Delta r_C$) will be chosen in the interval defined by $r_0 \equiv d_{blob}/2$, and $1.5\times 10^5 \le \Delta r_C \le 7\times 10^5$ cm, respectively;

- Particle densities, $N_O(r_O)$ and $N_C(r_C)$ in the volume of the sample of radius $R_{S,i}$. The sample volume will be considered spherical and its radius equal with the distance from the base of the wind at which the particle *i* (C, O or He) is injected in the supernova shock, $r_i \equiv R_{S,i}$. Also, the correlation length $r_0 < R_{S,i}$;

- The velocities $v_O(r_O)$, $v_C(r_C)$ and $v_{He}(r_{He})$.

Now, the differential wind equation in the assumptions made by Biermann et al. (1993) is:

$$\left(1-\frac{1}{y^3}\right)\frac{dy^2}{d(1/x)} = -g_{rad}\,, \tag{48}$$

with *x* the dimensionless length, *y* the velocity and g_{rad} the radiative acceleration (see (26)) in the radial streaming approximation (the results for an radiative acceleration with improved force multiplier (37) and exact depth parameter (35) will be presented in another paper).

Our "reduced" depth parameter will be:

$$\tilde{t} = (dv/dr)t\,, \tag{49}$$

with *t* from (27). So, the differential wind equation will look like:

$$\left(1-\frac{1}{y^3}\right)\frac{dy^2}{d(1/x)} = -\tilde{g}_{rad}\,\frac{dx}{dy}\,. \tag{50}$$

After doing the integration we find that:

$$y^2 - (1/y) = -\tilde{g}_{rad}(\ln x - x\ln x + x). \quad (51)$$

In the equation (51) the dimensionless value of $x \equiv r$/[cm]. Re-writing (51) for i element (O, C, or He) at $x = r_{in}$ and $x = r_{fin}$ ($r_{fin} = r_{in} + \Delta r$; $\Delta r \le r_0 = 7\times10^5$) distances, corresponding to $y_{in} \equiv v_{i,in}$ and $y_{fin} \equiv v_{i,fin}$, respectively, and making their substraction we get:

$$v_{i,fin}^2 - v_{i,in}^2 - \frac{1}{v_{i,fin}} + \frac{1}{v_{i,in}} = -\tilde{g}_{i,rad}\left(\ln r_{i,fin} - \ln r_{i,in} - r_{i,fin}\ln r_{i,fin} + r_{i,in}\ln r_{i,in} + r_{i,fin} - r_{i,in}\right). \quad (52)$$

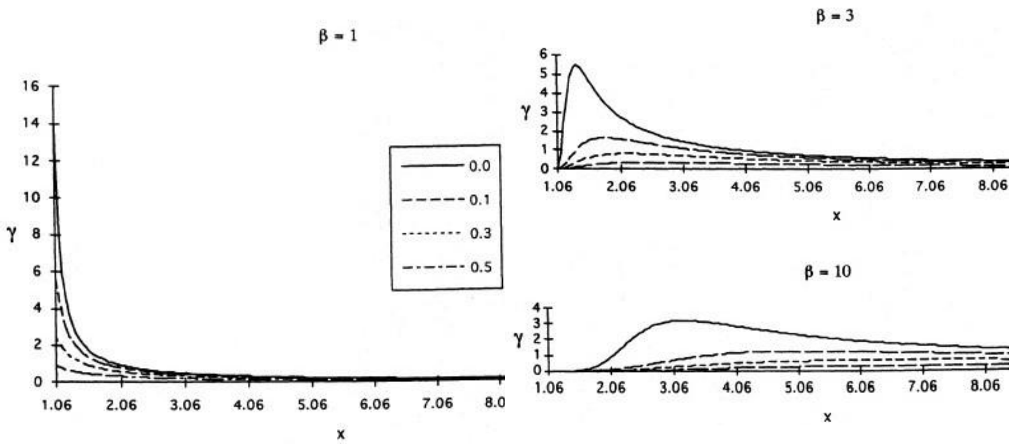

Fig.4 – Behavior of wind acceleration $\gamma = d(\ln v)/d(\ln r)$ as a function of distance xR for various values of parameters β, ε (the latter specified in the key) in wind velocity law $v(r) = v_\infty\left[\varepsilon + (1-\varepsilon)\left(1 - \frac{R_*}{r}\right)^\beta\right]$. (Brown et al. 1995)

Let now consider the case of two WR stars of equal masses, terminal velocities, and effective temperatures, one in WC pre-supernova sequence and the other in WO pre-supernova sequence. For this purpose we are using real stellar data from van der Hucht (2001), García-Segura et al. (1996), and Crowther et al. (1995). We suppose that in the wind of the WR star of WC spectral type is no O, but just He and C, while in the wind of the WR star of WO spectral type is no C (just He and O). This is allowed knowing that a WR star passes through both, WC and WO sequences, excepting the case when the star explodes as supernova before that, but even so, due to the fact that we are observing in CRs the contribution of all (and is no reason to think that the number of WR stars which explode in WC sequence is larger than for the ones that explode in WN sequence) WR stars as a unique spectral slope, this is allowed.

At a r_C for the WC sequence star we compute the velocity $v_C(r_C)$ (eq. (52)). With this velocity we "go" to the WO sequence star ($v_O^{'}(r_O^{'}) = v_C(r_C)$) and we compute $r_O^{'}$. Then, having $r_O^{'}$, we are "coming back" at the WC sequence star ($r_O = r_O^{'}$) and we compute the velocity $v_O(r_O)$. In this way, we will be able to write the equations for a WR star in the intermediary sequence WC/WO, in whose wind we find He, C and, also, O:

$$v_C^2 - v_{C,in}^2 - \frac{1}{v_C} + \frac{1}{v_{C,in}} = -\tilde{g}_{C,rad}\left(\ln r_C - \ln r_{C,in} - r_C \ln r_C + r_{C,in} \ln r_{C,in} + r_C - r_{C,in}\right), \tag{53}$$

$$v_O^2 - v_{O,in}^2 - \frac{1}{v_O} + \frac{1}{v_{O,in}} = -\tilde{g}_{O,rad}\left(\ln r_O - \ln r_{O,in} - r_O \ln r_O + r_{O,in} \ln r_{O,in} + r_O - r_{O,in}\right), \tag{54}$$

$$v_{He}^2 - v_{He,in}^2 - \frac{1}{v_{He}} + \frac{1}{v_{He,in}} = -\tilde{g}_{He,rad}\left(\ln r_{He} - \ln r_{He,in} - r_{He} \ln r_{He} + r_{He,in} \ln r_{He,in} + r_{He} - r_{He,in}\right), \tag{55}$$

where we consider that all the wind particles are originating at the stellar surface: $r_{O,in} = r_{C,in} = r_{He,in} = 1R_*$.

The fourth, the fifth, and the sixth equations of our system of seven equations will be:

$$v_O^2 - v_{O,in}^2 - \frac{1}{v_O} + \frac{1}{v_{O,in}} = -\tilde{g}_{O,rad}\left[\ln(r_C + b) - \ln r_{O,in} - (r_C + b)\ln(r_C + b) + r_{O,in} \ln r_{O,in} + (r_C + b) - r_{O,in}\right], \tag{56}$$

$$v_C^2 - v_{C,in}^2 - \frac{1}{v_C} + \frac{1}{v_{C,in}} = -\tilde{g}_{C,rad}\left[\ln r_C - \ln r_{C,in} - r_C \ln r_C + r_{C,in} \ln r_{C,in} + r_C - r_{C,in}\right], \tag{57}$$

$$v_{He}^2 - v_{He,in}^2 - \frac{1}{v_{He}} + \frac{1}{v_{He,in}} = -\tilde{g}_{He,rad}\left[\ln(r_C - a) - \ln r_{He,in} - (r_C - a)\ln(r_C - a) + r_{He,in} \ln r_{He,in} + (r_C - a) - r_{He,in}\right] \tag{58}$$

where $r_{He} = r_C - a$ and $r_O = r_C + b$. In the eq. (53) and (57) for the WC sequence WR star (see also (26) and (27)):

$$\tilde{g}_{C,rad} = \frac{1}{c\left(N_C(r_C) + N_{He}(r_{He})\right)} \sigma_{Th} \sigma_B T_{eff}^4 \left[k(\sigma_{Th} v_{th})^\alpha \right]. \tag{59}$$

Also, in (54) and (56) (for the WO sequence WR star):

$$\tilde{g}_{O,rad} = \frac{1}{c\left(N_O(r_O) + N_{He}(r_{He})\right)} \sigma_{Th} \sigma_B T_{eff}^4 \left[k(\sigma_{Th} v_{th})^\alpha \right], \tag{60}$$

and, in (55) and (58) (for the WC/WO sequence WR star):

$$\tilde{g}_{He,rad} = \frac{1}{c\left(N_O(r_O) + N_C(r_C) + N_{He}(r_{He})\right)} \sigma_{Th} \sigma_B T_{eff}^4 \left[k(\sigma_{Th} v_{th})^\alpha \right]. \tag{61}$$

In the approximation that all the WR wind particles are at least one time ionized, let us define a self-similar particle distribution (the ionic density in the in blobs). For this we must keep in mind (as seen in the previous sections) that the structure function study of wavelet images of the WR surrounding nebula is showing that there we have self-similar structures and that the power law of the energy dissipation is that of a compressible turbulence. The distribution of the number of structures (blobs), N, with the number of free electrons in a blob, N_e, reads (Richardson et al. 1996):

$$\frac{dN}{dN_e} \cong \frac{\Delta N}{\Delta N_e} = \frac{(1-\gamma) N_0}{N_{e,\max}^{1-\gamma} - N_{e,\min}^{1-\gamma}} N_e^{-\gamma}, \tag{62}$$

with N_0 - the total number of blobs at any time, $N_{e,\min}$ and $N_{e,\max}$ - the minimum and, respectively, the maximum number of free electrons contained in any blob: $N_{e,\max}$ = 10 $N_{e,\min}$, $N_{e,\max} = 2 \times 10^{46}$ (Richardson et al. 1996). Also, $\gamma = n - D$ (n - the space dimensionality = 3; D - the fractal dimension = 2.32).

In our case:

$$N_C(r_C) - N_{He}(r_{He}) = \frac{(1-\gamma)\left(\overline{N}_{0,C}(r_C) + \overline{N}_{0,He}(r_{He})\right)}{N_{e,\max}^{1-\gamma} - N_{e,\min}^{1-\gamma}} \left[N_e(r_C) - N_e(r_{He}) \right] \overline{N}_e^{-\gamma} \tag{63}$$

$$N_O(r_O) - N_{He}(r_{He}) = \frac{(1-\gamma)\left(\overline{N}_{0,O}(r_O) + \overline{N}_{0,He}(r_{He})\right)}{N_{e,\max}^{1-\gamma} - N_{e,\min}^{1-\gamma}} \left[N_e(r_O) - N_e(r_{He}) \right] \overline{N}_e^{-\gamma} \tag{64}$$

where the equations (63) and (64) are for our WR star in WC sequence and, respectively, in WO sequence, when we consider that in the wind the spatial separation of C from He (WC case) and of O from He (WO case) is giving an self-similar particle distribution in blobs (each particle type is forming its own self-similar like blob structures). In the right side of (63) and (64), $\overline{N}_{0,C}(r_C)$, $\overline{N}_{0,O}(r_O)$, $\overline{N}_{0,He}(r_{He})$ are the average C, O and He densities, and we can find them from the matter conservation law:

$$\dot{M}_i = 4\pi \overline{\rho}_i(r_i) r_i^2 v_{esc}. \tag{65}$$

Then, the average density $\overline{\rho}_i(r_i)$ (in g/cm^3) for *i* wind element over the sample (blob) volume:

$$\overline{\rho}_i(r_i) = \dot{M}_i / \left(4\pi r_i^2 v_{esc}\right), \tag{66}$$

and if we consider that the total mass loss, $\dot{M}(X_j)$ ($j \in [1, i]$) of each stellar surface element isotope (Woosley et al. 1995) (assuming that the surface mass fraction X_j is having a small time dependency - in the supernova deflagration time scale, comparatively with the stellar age):

$$\dot{M}(X_j) = X_{j,surf}(t)\dot{M} \cong X_{j,surf}\dot{M}, \tag{67}$$

results that:

$$\overline{\rho}_i(r_i) = \left(\dot{M}X_i\right) / \left(4\pi r_i^2 v_{esc}\right). \tag{68}$$

In (68), $X_i = \sum_{j=1}^{i} X_j$ and the mass loss for Wolf-Rayet hydrogen depleted WC/WO pre-supernova stars with initial masses between 30-35 $M_\odot$ and 50 $M_\odot$ is described through the empirical formula (Langer 1989b and Woosley et al. 1993):

$$\dot{M} = 10^{-7} \times (M/M_\Theta)^{2.6} \quad [M_\Theta / yr]. \tag{69}$$

The X_j surface mass fractions are the Langer's et al. stellar evolution computation results (Langer et al. 1995) (see table 2).

Table 2

Surface mass fractions of various isotopes in stellar evolution models in the initial mass range $30M_\odot \le M \le 50M_\odot$ at the pre-SN state. The pre-SN configuration is also indicated, where RSG means red supergiant and WC stands for Wolf-Rayet star of the carbon sequence. (Langer et al. 1995)

Isotop	$30M_{\odot}$ RSG → WC	$40M_{\odot}$ WC	$50M_{\odot}$ WC
^{1}H	6.16×10^{-1}	0.0	0.0
^{4}He	3.65×10^{-1}	7.22×10^{-1}	1.49×10^{-1}
^{12}C	2.01×10^{-3}	2.07×10^{-1}	4.94×10^{-1}
^{13}C	1.39×10^{-4}	4.70×10^{-7}	0.0
^{14}N	4.83×10^{-3}	4.51×10^{-3}	0.0
^{15}N	1.62×10^{-6}	0.0	0.0
^{16}O	7.42×10^{-3}	4.81×10^{-2}	3.32×10^{-1}
^{17}O	6.95×10^{-5}	8.33×10^{-6}	0.0
^{18}O	8.22×10^{-6}	8.92×10^{-4}	0.0

Since the average particle number density:

$$\overline{N}_{0,i}(r_i)=\overline{\rho}_i(r_i)/\overline{m}=\overline{\rho}_i(r_i)/(\mu\, m_H), \tag{70}$$

where the mean molecular weight $\mu \equiv \overline{m}/m_H$ (with $\overline{m}$, the average mass of the gas particle and $m_H = 1.673525\times10^{-24}$ [g], the mass of a hydrogen atom) and $\mu_{ion} < \mu < \mu_{neu}$ (μ_{ion} and μ_{neu} are the mean molecular weights for the completely ionized and for completely neutral gas, respectively) (Carroll et al. 1996):

$$\frac{1}{\mu_{ion}} \cong 2X_H + \frac{3}{4}X_{He} + \left\langle \frac{1+z}{A} \right\rangle_{ion} \left(\sum_j X_j - X_H - X_{He} \right) \cong 2X_H + \frac{3}{4}X_{He} + \frac{1}{2}\left(\sum_j X_j - X_H - X_{He} \right) \tag{71}$$

$$\frac{1}{\mu_{neu}} \cong X_H + \frac{1}{4}X_{He} + \left\langle \frac{1}{A} \right\rangle_{neu} \left(\sum_j X_j - X_H - X_{He} \right) \cong X_H + \frac{1}{4}X_{He} + \frac{1}{15.5}\left(\sum_j X_j - X_H - X_{He} \right) \tag{72}$$

we will be able to write, with the help of eq. (68):

$$\overline{N}_{0,i}(r_i) = \left(\dot{M}X_i\right) / \left(4\pi r_i^2 v_{esc}\, \mu m_H\right). \tag{73}$$

Also, in (63) and (64):

$$\overline{N}_{0,i}(r_i) + \overline{N}_{0,He}(r_{He}) = \left[\left(\overline{N}_{0,i}(r_i)/\overline{N}_{0,He}(r_{He})\right)+1\right]\overline{N}_{0,He}(r_{He}). \tag{74}$$

Observing from (73) that:

$$\overline{N}_{0,i}(r_i)/\,\overline{N}_{0,He}(r_{He}) = (X_i\,/\,X_{He})\cdot(m_{He}\,/\,m_i), \tag{75}$$

we find that:

$$\overline{N}_{0,i}(r_i) + \overline{N}_{0,He}(r_{He}) = [(X_i\,/\,X_{He})\cdot(m_{He}\,/\,m_i)+1]\overline{N}_{0,He}(r_{He}), \tag{76}$$

where the values of X_i are the sum of the isotopic mass fraction from table 2 for C or O, and m_i and m_{He} are the atomic masses of *i* element and the atomic mass of He, respectively, expressed in grams.

Now, if we consider that the free electrons in the WR wind are catched in the same instability driven turbulent general motion as the ions and that they have a self-similar distribution in the blobs and a random distribution between blobs, we have from (21):

$$N_e(r_i) = \overline{N}_e\left[f_{i,e}^2 A_{i,bb} r_i^{-\gamma} + 1 - f_{i,e}^2\right] \quad , \quad f_{i,e} < 1, \tag{77}$$

with (see (24)):

$$f_{i,e} = \frac{K_i R_{i,S}^D}{K_i R_{i,S}^D + K_i^{'} R_{i,S}^3}\,, \tag{78}$$

$$A_{i,bb} = A_{i,ee}\,/\,f_{i,e}^2\,, \tag{79}$$

and (see (25)):

$$A_{i,ee} = \left(1-\frac{\gamma}{3}\right)\frac{R_{i,S}^{-\gamma}}{R_{i,S}^{-2\gamma} + \left(\frac{K'}{K}\right)_i^2 + 2R_{i,S}^{-\gamma}\left(\frac{K'}{K}\right)_i} \equiv r_{i,0}^{\gamma}\,, \tag{80}$$

we get a full description of the densities that enter in the right side of (63) and (64) by making the (76) and (77), and (from (73)):

$$\overline{N}_{0,He}(r_{He}) = \frac{\dot{M} X_{He}}{4\pi r_{He}^2 v_{esc}\,\mu m_H}\,, \tag{81}$$

substitutions. In the above equation v_{esc} is given by (34). For $\overline{N}_e$ we take the average value between $N_{e,\min}$ and $N_{e,\max}$ over the volume of radius $R_{i,S}$.

After substitution, one last problem arises in (63) and (64) by the apparition of the $(K'/K)_i$ ratios. This can be solved considering that the observed blobs are having a universal fractal distribution where the fractal behavior $\zeta(p)$ for the p order (which can be regarded as a spatial scaling of the analyzed image) of the structure function. Because also the correlation length $r_{i,0}$ for the i element in blobs is (like $\zeta(p)$ but with a much smaller generality) a measure of fractality in the system and because $R_{i,S}$ is for certain a spatial scaling measure, we postulate that (see also the similarity between Fig. 1 and Fig. 2) in three dimensional space:

$$\zeta(p)/p \propto r_{i,0}/R_{i,S}, \tag{82}$$

and, in a particular case:

$$\zeta(p)/p \cong r_{i,0}/R_{i,S}. \tag{83}$$

With $r_{i,0}$ from (80) and with (10) in (83) we find:

$$(K'/K)_i = R_{i,S}^{-\gamma}\left[-1+\sqrt{1-(\gamma/3)}(\zeta(p)/p)^{-\frac{\gamma}{2}}\right]. \tag{84}$$

Subtracting (64) from (63) we get the seventh equation of our system of equations for a WC/WO Wolf-Rayet star without companion.

4. RESULTS AND CONCLUSIONS

Even if it is a little disappointing for the reader, our primal purpose was to show that, the system of equations constructed on the above assumptions converges to a solution for any value of the $r \equiv r_C$ in the range 1-3 R_*. Our numerical code, made in IDL 5.2, was tested for convergence when we independently varied quantities like the stellar initial mass ($30M_\odot \le M \le 50M_\odot$; with the corresponding effective temperatures, terminal velocities, mass fractions, etc.), molecular weight ($\mu_{ion} < \mu < \mu_{neu}$), ε ($v_{th} = \varepsilon v_\infty$; 0.01 ≤ ε ≤ 0.1), or iteration step $\Delta r \equiv \Delta r_C$ (when $r_C \to r_C + \Delta r_C$; $1.5\times10^5 \le \Delta r_C \le 7\times10^5$). Our unique model free parameter, p ($0.1 < p \le 3$), had also the not to be ignored purpose to enable a rescaling of our computational lattice in such a way that the sample radius, $R_{i,S}$, never to touch the lattice border which would had meant a disaster ("explosion") for the system variables.

Indeed, the system converged in all the above situations, which meant that we had an pre-supernova phase space dispersion between C, O, and He in the wind, seen in the observed CRs through the factor $(Z_i/Z_{He})^{k_i}$, and this is done by the assumption that, prior to the acceleration at the terminal shock, the ions are radiative accelerated.

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