ORBIT CHANGES IN MARS' LOWER ATMOSPHERE

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Abstract. Using Sehnal's exponential law to describe the density distribution in the Mars' atmosphere at heights up to 100 km, the first order drag perturbations in the motion of an orbiter evolving in this height range are analytically determined over one nodal period. Mars' oblateness and its atmospheric rotation are considered. The analytic approach of passive trajectories at such altitudes is justified.

Key words: Mars' Atmoshere — Orbital Motion

1. INTRODUCTION

Using the numerical data listed by the MA—87 model (Moroz et al. 1988), Sehnal (1990 a, b) approximated the height-dependence of the Martian atmospheric density by analytic formulae for two altitude ranges: 100—1000 km and up to 100 km (in both cases for three density profiles: minimal, nominal, and maximal). The density formula for 100—1000 km was used by Sehnal (1990 a) to find some features of a Martian orbiter motion, and by us (e.g. Mioc et al. 1991) to perform a similar (but more general from several viewpoints) study.

For altitudes up to 100 km, Sehnal (1990 b) proposed a parabolic dependence of $\ln \rho$ on h (ρ = density at the height h):

$$\ln(\rho/\rho_0) = \sum_{j=0}^{2} a_j h^j, \tag{1}$$

where ρ and $\rho_0 = 1$ are expressed in kg/m^3, h is given in km, while the coefficients a_j are separately determined for the minimal, nominal, and maximal density profiles. He improved this formula by considering a_j as asymmetric terms depending on the latitude φ:

$$\rho = \rho_0 \exp\left(\sum_{j=0}^{2} (a_j + a_{j1} \sin\varphi)\, h^j\right), \tag{2}$$

with (nominal model): $a_{00} = -4.1235$, $a_{01} = -0.1206$, $a_{10} = -0.0930$, $a_{11} = 0.0136$, $a_{20} = -0.00023$, $a_{21} = -0.00011$.

Of course, since the lifetime of a presumptive orbiter (in passive trajectory) at so low altitudes is very short, the study of its motion seems to be of small interest for practical purposes (especially for manned Martian space missions). However, for reasons we shall expose in Section 5, in this paper we shall estimate analytically the perturbations caused by the atmospheric drag in five independent orbital parameters :

$$z \in Y = \{p,\ q = e\ \cos\ \omega,\ k = e\ \sin \omega,\ \Omega,\ i\} \tag{3}$$

of a Mars' orbiter (in an initial orbit entirely contained in the height range 0—100 km) over one nodal period. Here p = semilatus rectum, e = eccentricity, ω = argument of periastron, Ω = longitude of ascending node, i = inclination (all with respect to the classical planetocentric frame). The Mars' oblateness ($\varepsilon = 0.005$) and its atmospheric rotation (of constant angular velocity w) will also be considered.

2. EQUATIONS OF MOTION

Since we study the motion over one nodal period, we adopt as independent variable the argument of latitude (u). The corresponding Newton-Euler system, from which we start, will be used under the form (Mioc 1991) :

$$\begin{aligned}
&\mathrm{d}p/\mathrm{d}u = 2(Z/\mu)r^3T,\\
&\mathrm{d}q/\mathrm{d}u = (Z/\mu)(r^3kBCW/(pD) + r^2T(r(q + A)/p + A) + r^2BS),\\
&\mathrm{d}k/\mathrm{d}u = (Z/\mu)(-r^3qBCW/(pD) + r^2T(r(k + B)/p + B) - r^2AS),\\
&\mathrm{d}\Omega/\mathrm{d}u = (Z/\mu)r^3BW/(pD),\\
&\mathrm{d}i/\mathrm{d}u = (Z/\mu)r^3AW/p,\\
&\mathrm{d}t/\mathrm{d}u = Zr^2(\mu p)^{-1/2},
\end{aligned} \tag{4}$$

where $Z = 1/(1 - r^2C\Omega/(\mu p)^{1/2})$, μ = Mars' gravitational parameter, r = planetocentric radius vector, $A = \cos u$, $B = \sin u$, $C = \cos i$, $D = \sin i$, while S, T, W are, respectively, the radial, transversal, and binormal components of the perturbing acceleration.

With initial h smaller than 100 km and R (Mars' equatorial radius) = 3393.4 km (Moroz et al. 1988), the initial e does not exceed 0.015 ; we shall therefore consider only quasi-circular orbits (expansions to first order in q and k). So, from the well-known orbit equation in polar coordinates, $r = p/(1 + e \cos v) = p/(1 + Aq + Bk)$, where v = true anomaly, one will write :

$$r^n = p^n(1 - nAq - nBk). \tag{5}$$

Also, since the perturbing factor is the atmospheric drag, we shall have in the same conditions (Mioc 1991) :

$$\begin{aligned}
&S = -\rho\delta(\mu/p)(Bq - Ak),\\
&T = -\rho\delta(\mu/p)(1 + 2Aq + 2Bk) + \rho\delta Cw(\mu p)^{1/2},\\
&W = -\rho\delta Dw(\mu p)^{1/2}A,
\end{aligned} \tag{6}$$

where δ is the drag parameter of the orbiter.

We shall integrate (4) by successive approximations, with $Z \cong 1$, limiting the process to first order approximation. So, we may consider separately the first five equations (4). With (5) and (6), and omitting in advance the factor Z, these equations acquire the form:

$$\begin{aligned}
&\mathrm{d}p/\mathrm{d}u = pb(2(x+y) - 2(3x+y)Aq - 2(3x+y)Bk)\rho,\\
&\mathrm{d}q/\mathrm{d}u = b(2(x+y)A + ((x+2y) - (5x+2y)A^2)q - 2(3x+y)ABk)\rho,\\
&\mathrm{d}k/\mathrm{d}u = b(2(x+y)B - 2(2x+y)ABq + (-4x+(5x+2y)A^2)k)\rho,\\
&\mathrm{d}\Omega/\mathrm{d}u = b(x/C)(-AB + 3A^2Bq + 3AB^2k)\rho,\\
&\mathrm{d}i/\mathrm{d}u = b(Dx/C)(-A^2 + 3A^3q + 3A^2Bk)\rho,
\end{aligned} \tag{7}$$

where we used the abbreviating notation:

$$b = p^{5/2}\mu^{-1/2}\delta, \; x = Cw, \; y = -p^{-3/2}\mu^{1/2}. \tag{8}$$

The variations of the orbital elements (3) over one nodal period will be:

$$\Delta z = \int_0^{2\pi} (\mathrm{d}z/\mathrm{d}u)\mathrm{d}u, \; z \in Y, \tag{9}$$

with the integrands given by (7). In order to write these equations in a suitable form for performing the integrals (9), we still have to express the density as function only of u (through A and B).

3. EXPRESSION OF THE DENSITY

According to our purposes, we shall adopt for the density distribution law (2). Firstly we consider the altitude:

$$h = r - R(1 - \varepsilon \sin^2\varphi). \tag{10}$$

With r given by (5), and with:

$$\sin\varphi = DB, \tag{11}$$

this formula becomes:

$$h = p - R + \varepsilon RD^2B^2 - pAq - pBk. \tag{12}$$

Now we introduce (11) and (12) into (2), and expand the exponential to first order in q and k. In this expansion, the exponential is kept for the constant terms, while for the variable ones (those containing A

and B), which are small, only two terms will be kept in expansion, except for the quantities $a_{11}(p-R)DB$ and $a_{21}(p-R)^2DB$, which are of the order of unity; for these ones the exponential will be separately expanded, keeping four terms in the respective expansions. Acting in the specified way, and introducing successively the abbreviating notation:

$$X_0 = \rho_0 \exp\left(\sum_{j=0}^{2} a_{j0}(p-R)^j\right); \tag{13}$$

$$c_0 = 1,\ c_1 = a_{01}D,\ c_2 = \varepsilon RD^2(a_{10} + 2a_{20}(p-R)),$$
$$c_3 = \varepsilon RD^3(a_{11} + 2a_{21}(p-R)),\quad c_4 = a_{20}\varepsilon^2R^2D^4,\ c_5 = a_{21}\varepsilon^2R^2D^5; \tag{14}$$

$$d_0 = 1,\ d_1 = D(p-R)(a_{11} + a_{21}(p-R)),\ d_2 = d_1^2/2,\ d_3 = d_1^3/6,$$
$$d_4 = D^4(p-R)^5a_{11}a_{21}(a_{11}^2/6 + a_{11}a_{21}(p-R)/4 + a_{21}^2(p-R)^2/6),$$
$$d_5 = D^5(p-R)^7a_{11}^2a_{21}^2(a_{11} + a_{21}(p-R))/12, \tag{15}$$
$$d_6 = D^6a_{11}^3a_{21}^3(p-R)^9/36;$$

$$Q_0 = -p(a_{10} + 2a_{20}(p-R)),\ Q_1 = -pD(a_{11} + 2a_{21}(p-R)),$$
$$Q_2 = -2a_{20}\varepsilon pRD^2,\ Q_3 = -2a_{21}\varepsilon prD^3; \tag{16}$$

$$L_m = \sum_{i+j=m} c_i d_j,\ i = \overline{0,5},\ j = \overline{0,6}; \tag{17}$$

$$R_t = \sum_{n+n=t} L_mQ_n,\ m = \overline{0,11},\ n = \overline{0,3}, \tag{18}$$

the expression of the density becomes:

$$\rho = X_0\left(\sum_{m=0}^{11} L_mB^m + (Aq + Bk)\sum_{t=0}^{14} R_tB^t\right). \tag{19}$$

4. VARIATIONS OF THE ORBITAL ELEMENTS

By (19), the density is expressed only in terms of A, B, and quantities considered constant over one revolution. Substituting (19) into (7), the motion equations become:

$$\mathrm{d}p/\mathrm{d}u = X_0pb(P_0S_1 + (P_0S_2 + P_1S_1)Aq + (P_0S_2 + P_1S_1)Bk),$$

$$\mathrm{d}q/\mathrm{d}u = X_0b(P_0AS_1 + (P_2S_1 + (P_0S_2 + P_3S_1)A^2)q + (P_0S_2 +$$
$$+ P_1S_1)ABk),$$

$$\mathrm{d}k/\mathrm{d}u = X_0 b(P_0 BS_1 + (P_0 S_2 + P_4 S_1)ABq + (P_5 S_1 + P_3 A^2 S_1 + + P_0 B^2 S_2)k), \tag{20}$$

$$\mathrm{d}\Omega/\mathrm{d}u = X_0 b(x/C)(-ABS_1 + (3S_1 - S_2)A^2 Bq + (3S_1 - S_2)AB^2 k),$$

$$\mathrm{d}i/\mathrm{d}u = X_0 b(Dx/C)(-A^2 S_1 + (3S_1 - S_2)A^3 q + (3S_1 - S_2)A^2 Bk),$$

where we used the notation:

$$S_1 = S_1(B) = \sum_{m=0}^{11} L_m B^m, \ S_2 = S_2(B) = \sum_{l=0}^{14} R_l B^l; \tag{21}$$

$$P_0 = 2(x + y), \ P_1 = -2(3x + y), \ P_2 = x + 2y,$$

$$P_3 = -(5x + 2y), \ P_4 = -2(2x + y), \ P = -4x. \tag{22}$$

Finally, performing the integrals (9) with the integrands given by (20), we obtain the perturbations of the orbital elements over one nodal period:

$$\Delta p = 2\pi X_0 pb \sum_{n=0}^{7} f_n (P_0 L_{2n} + (P_1 L_{2n-1} + P_0 R_{2n-1})k),$$

$$\Delta q = 2\pi X_0 bq \sum_{n=0}^{8} f_n (P_5 L_{2n} - P_3 L_{2n-2} + P_0 (R_{2n} - R_{2n-2})),$$

$$\Delta k = 2\pi X_0 b \sum_{n=0}^{8} f_n (P_0 L_{2n-1} + ((P_3 + P_5) L_{2n} - P_3 L_{2n-2} + P_0 R_{2n-2})k), \tag{23}$$

$$\Delta\Omega = 2\pi X_0 b(x/C) q \sum_{n=1}^{8} f_n (3(L_{2n-1} - L_{2n-3}) - (R_{2n-1} - R_{2n-3})),$$

$$\Delta i = 2\pi X_0 b(Dx/C) \sum_{n=0}^{8} f_n (L_{2n2} - L_{2n} + (3(L_{2n-1} - L_{2n-3}) - - (R_{2n-1} - R_{2n-3}))k),$$

where we introduced artificially $L_{-i} = R_{-i} = 0$ $(i = \overline{1,3})$, $L_j = 0$ $(j = \overline{12, 16})$, $R_k = 0$ $(k = 15, 16)$, and wrote

$$f_0 = 1, \ f_n = ((2n-1)!!)/(2^n n!), \ n \in \mathbf{N}^*. \tag{24}$$

Of course, various particular cases, as for instance initially circular orbit ($q = k = 0$), or neglection of atmospheric rotation ($x = 0$), can also be studied operating in the above formulae the corresponding modifications.

5. COMMENTS

Consider a body in passive trajectory orbiting Mars such that $h \lesssim 100$ km. Using our final formulae, we estimated numerically that, because of the strong atmospheric drag at such altitudes, the respective body cannot perform but few revolutions before falling on Mars' surface. Moreover, in order to accomplish some passive orbits, the body must have a very small drag parameter (it must be very massive and of small dimensions). It seems to follow that an analytic study of the passive orbits in the Mars' atmosphere at heights up to 100 km would be of small interest for concrete situations (especially for practical purposes related to space dynamics).

Nevertheless we consider that such a study is of both practical and theoretical interest. From the practical point of view, the knowledge of the drag perturbations on the passive trajectory can provide useful data for determining the orbit corrections to be performed by manoeuvres. It is also useful to know how long a presumptive Martian lander can fly on passive orbit lower than 100 km.

From a theoretical point of view, although generated by a concrete case (orbits in the Mars' atmosphere at heights up to 100 km), our research can constitute an analytic study of the orbital motion in a resisting medium whose density distribution is described by the law (2), with the coefficients a_0, a_1 of the same order of magnitude, but covering a larger height range. Another problem to which our theoretical results can be applied is that of the purely dynamically perturbed motion of a meteoroid-like body, captured by Mars, whose circummartian orbit brought it in the above considered atmospheric layer.

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