MATHEMATICAL ALGORITHM FOR THE DARK-FLIGHT TRAJECTORY OF A METEOROID

IOANA BOACA[1], ALIN NEDELCU[1,2], MIREL BIRLAN[2,1],
TUDOR BOACA[3,4], SIMON ANGHEL[1,2,5]

[1] *Astronomical Institute of Romanian Academy,*
Str. Cutitul de Argint 5, 040557 Bucharest, Romania
Email: ioana.boaca@astro.ro
[2] *IMCCE, Observatoire de Paris*
77 av Denfert Rochereau, 75014 Paris cedex, France
[3] *Department of Computer Science, Information Technology, Mathematics and Physics,*
Petroleum-Gas University of Ploiesti,
39 Bucharest Street, 100680, Ploiesti, Romania
[4] *Simion Stoilow Institute of Mathematics of the Romanian Academy,*
21 Calea Grivitei Street, 010702 Bucharest, Romania
[5] *Faculty of Physics, University of Bucharest*
405-Atomistilor, 077125 Magurele, Ilfov, Romania

Abstract. In this paper we propose a new mathematical model for the dark-flight trajectory of a meteoroid. We take into account the influence of the wind, the properties of the atmosphere, the Coriolis force and the centrifugal force. This model uses the ellipsoid shape of the Earth instead of the classical spherical one. The components of the wind are treated using a Monte Carlo simulation. A numerical example for the algorithm is also presented.

Key words: Meteoroid – Meteor – Meteorite – Dark-flight.

1. INTRODUCTION

The paper presents a mathematical model for the dark-flight trajectory of a meteoroid. The subject of dark-flight is defined and well studied in the literature (Moilanen, Gritsevich, and Lyytinen, 2021; Vinnikov, Gritsevich, and Turchak, 2017, 2016; Kuznetsova, Gritsevich, and Vinnikov, 2014; Gritsevich *et al.*, 2014; Kuznetsova and Gritsevich, 2014; Tóth *et al.*, 2015). The dark-flight of the meteoroid is important in order to determine the search area for the fragments reaching the surface of the Earth. This area is more commonly named *strewn field* by meteoriticists. More generally, the dark flight is interesting for both natural and artificial bodies which are falling into the Earth atmosphere. Computation of ballistics for space missions which release payloads on Mars are also getting in touch with such topics.

The following terminology will be used in this paper (Silber *et al.*, 2018):

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- *Meteoroids* are objects of asteroidal or cometary origin that have smaller size * than asteroids that orbit the Sun.
- The *meteor* is the luminous phenomenon generated by a meteoroid entering the atmosphere.
- After the luminous phenomenon ends the object passes through the atmosphere; this part of the trajectory is called *dark-flight*.
- The *meteorite* is the remnant matter that reaches the ground.

Several factors are influencing the fall of meteoroid during the dark flight movement (Borovicka and Kalenda, 2003; Brown *et al.*, 2004). Among the most important we underline the atmospheric influence and the Coriolis and centrifugal effects of non-inertial systems. For instance, the dark-flight trajectory is dependent on the properties of the atmosphere and on the wind profile. The procedure used by (Borovicka and Kalenda, 2003) takes into account atmospheric drag, horizontal winds, and Earth's gravity in the assumption of the meteoroid as spherical body. This similar procedure is also used by several authors (*e.g.* Brown *et al.* (2004); Jeanne *et al.* (2019)). However, models of atmosphere do not take into account the whole wind profile, Earth's real ellipsoidal shape.

In the following sections we will describe the mathematical theory for the dark-flight trajectory of a single body and we will present the atmospheric models used in order to determine the density, temperature, and pressure of the atmosphere.

The structure of this paper is the following: in Section 2 the equations and the forces that determine the free fall of an object are presented, Section 3 presents the dark flight trajectory of the meteoroid taking into account the Coriolis effect and the centrifugal force. Section 4 describes the main atmospheric models for Earth. Section 5 presents an example of dark-flight trajectory and the influence of various parameters on the dark-flight trajectory..

2. MODEL FOR THE FREE FALL OF AN OBJECT

One of the algorithms used in the theory of free fall was proposed by Biringen and Chow (2011). In this approach only two components of movement were considered. In what follows the 3D case of this theory is considered and the equation of motions for the three components are computed.

In what follows a spherical object of mass m and radius r_b that enters the atmosphere is considered in a 3D reference system.

*The range of transition between asteroids and meteoroids is considered between 1 and 10 meters in diameter.

Let $\{O;X,Y,Z\}$ a Cartesian coordinate system, with the OZ axis vertical. For the large part of problems related to the fall into an atmosphere of a planet, the support of vertical axis OZ could be associated to the lines of gravitational field.

In order to obtain the equations of movement of a body of mass m through a fluid we will take into account the force of inertia, the Buoyancy force and the friction forces between the atmosphere and the body.

Moreover, in the study of the movement of an object that accelerates through a fluid we will have to take into account the force of inertia of the fluid outside the body, fluid that must be accelerated. When writing the equations of movement we will take into account this force by adding a supplementary mass m' to the mass of the body, the so-called 'added mass' (Biringen and Chow, 2011; Lamb, 1993).

Let $\mathbf{f} = (f_X, f_Y, f_Z)$ the friction force between the body and the atmosphere (drag force), and m_f the mass of fluid displaced by the body. Starting from Newton's second law we obtain the movement equations of the body under the form:

$$\begin{cases} \left(m+m'\right)\frac{\mathrm{d}^2X}{\mathrm{d}t^2} = f_X \\ \left(m+m'\right)\frac{\mathrm{d}^2Y}{\mathrm{d}t^2} = f_Y \\ \left(m+m'\right)\frac{\mathrm{d}^2Z}{\mathrm{d}t^2} = -(m-m_f)g + f_Z \end{cases} \tag{1}$$

where m_f is the mass of the fluid displaced by the sphere and g is the gravitational acceleration.

In the case of a sphere (Lamb, 1993) theoretically obtained using the potential flow theory that $m' = \frac{1}{2}m_f$.

Let $\mathbf{W}_f = (u_f, v_f, w_f)$ and $\mathbf{W} = \left(\frac{\mathrm{d}X}{\mathrm{d}t}, \frac{\mathrm{d}Y}{\mathrm{d}t}, \frac{\mathrm{d}Z}{\mathrm{d}t}\right)$ the fluid velocity and the body velocity, respectively. For the drag force we use the following formula (Vinnikov, Gritsevich, and Turchak, 2016):

$$\mathbf{f} = 0.5\pi\rho_f r_b^2 c_d |\mathbf{W}_f - \mathbf{W}| (\mathbf{W}_f - \mathbf{W}) \tag{2}$$

where ρ_f is the fluid density and c_d is the friction coefficient.

Thus equations (1) can be written under the form :

$$\begin{cases} \frac{\mathrm{d}^2X}{dt^2} = \frac{3}{8}\frac{\overline{\rho}}{1+0.5\overline{\rho}}\frac{1}{r_b}c_d \mid \mathbf{W}_f - \mathbf{W} \mid (u_f - \frac{\mathrm{d}X}{\mathrm{d}t}) \\ \frac{\mathrm{d}^2Y}{\mathrm{d}t^2} = \frac{3}{8}\frac{\overline{\rho}}{1+0.5\overline{\rho}}\frac{1}{r_b}c_d \mid \mathbf{W}_f - \mathbf{W} \mid (v_f - \frac{\mathrm{d}Y}{\mathrm{d}t}) \\ \frac{\mathrm{d}^2Z}{\mathrm{d}t^2} = -\frac{1-\overline{\rho}}{1+0.5\overline{\rho}}g + \frac{3}{8}\frac{\overline{\rho}}{1+0.5\overline{\rho}}\frac{1}{r_b}c_d \mid \mathbf{W}_f - \mathbf{W} \mid (w_f - \frac{\mathrm{d}Z}{\mathrm{d}t}) \end{cases} \tag{3}$$

where $\overline{\rho} = \frac{\rho_f}{\rho}$, and ρ is the density of the body.

In Section 3 instead of system (3) we obtain another differential system for the study of the dark-flight trajectory, namely we neglect the influence of the inertial force of the fluid (this force is very low in the case when a meteoroid moves in the air due to the fact that the density of air is much lower than the density of the meteoroid) and we take into account the Coriolis effect.

3. DARK-FLIGHT TRAJECTORY OF METEOROID ENTERING EARTH'S ATMOSPHERE

In this section we present the mathematical model of dark-flight trajectory of a meteoroid. In the general equations (3) of Section 2 we neglect the inertial force of the fluid, thus we use the universal law of attraction and we consider the Coriolis effect. The inertial force could be neglected due to the fact that it induces a very low value into the equation.

Let $\{O;x_1,y_1,z_1\}$ an inertial Cartesian coordinate system and $\{O;x,y,z\}$ a non-inertial Earth-fixed Cartesian system with the associated versors $\mathbf{i}_1$,$\mathbf{j}_1$,$\mathbf{k}_1$ and $\mathbf{i}$,$\mathbf{j}$,$\mathbf{k}$, respectively. The origin of the two coordinate system is taken in the center of the Earth. The Ox_1, Oy_1, Ox, Oy axes are situated in the Equatorial plane. The Ox axis is directed towards the Greenwich meridian.

The non-inertial system $\{O;x,y,z\}$ rotates around the Oz axis with the angular velocity Ω (Earth's angular velocity).

In order to determine the equations of movement of the meteoroid in the non-inertial frame we start from the second law of Newton which in the inertial frame is written under the form:

$$m\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} = \mathbf{F} + \mathbf{F}_f \tag{4}$$

where

$$\mathbf{r} = x_1\mathbf{i}_1 + y_1\mathbf{j}_1 + z_1\mathbf{k}_1 = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

and

$$r = |\mathbf{r}| = \sqrt{x^2+y^2+z^2} = \sqrt{x_1^2+y_1^2+z_1^2}$$

(the geocentric distance), $\mathbf{F}$ is the gravitational force between Earth and the meteoroid and $\mathbf{F}_f$ is the friction force between the atmosphere and the meteoroid and m is the mass of the meteoroid.

In order to determine the gravitational force $\mathbf{F}$, we use Newton's law of universal gravitation and we obtain:

$$\mathbf{F} = -kM\frac{\mathbf{r}}{r^3} \tag{5}$$

where k is the gravitational constant and M is the mass of the Earth.

In order to calculate the friction force $\mathbf{F}_f$ we use formula (2).

Let $\mathbf{a} = (\frac{\mathrm{d}^2x_1}{\mathrm{d}t^2}, \frac{\mathrm{d}^2y_1}{\mathrm{d}t^2}, \frac{\mathrm{d}^2z_1}{\mathrm{d}t^2})$ the acceleration of the meteoroid in the inertial frame, $\mathbf{a}_r = (\frac{\mathrm{d}^2x}{\mathrm{d}t^2}, \frac{\mathrm{d}^2y}{\mathrm{d}t^2}, \frac{\mathrm{d}^2z}{\mathrm{d}t^2})$ the acceleration of the meteoroid in the non-inertial frame and $\mathbf{v}_r$ the velocity of the meteoroid in the non-inertial frame.

Taking into account the Coriolis effect and the centrifugal force we obtain:

$$\mathbf{a} = \mathbf{a}_r + 2\mathbf{\Omega} \times \mathbf{v}_r + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}) \tag{6}$$

where

$$\mathbf{\Omega} = \Omega\mathbf{k} = \Omega\mathbf{k}_1 \tag{7}$$

From (4), (5) and (6) we obtain the following non-linear differential system modelling the meteoroid entering the atmosphere of the Earth:

$$\begin{cases} \frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\mu\frac{x}{r^3} + \frac{3}{8}c_d\overline{\rho}\frac{1}{r_b} \mid \mathbf{W}_f - \mathbf{W} \mid (u_f - \frac{\mathrm{d}x}{\mathrm{d}t}) + \Omega^2 x + 2\Omega\frac{\mathrm{d}y}{\mathrm{d}t} \\ \frac{\mathrm{d}^2y}{\mathrm{d}t^2} = -\mu\frac{x}{r^3} + \frac{3}{8}c_d\overline{\rho}\frac{1}{r_b} \mid \mathbf{W}_f - \mathbf{W} \mid (v_f - \frac{\mathrm{d}y}{\mathrm{d}t}) + \Omega^2 y - 2\Omega\frac{\mathrm{d}x}{\mathrm{d}t} \\ \frac{\mathrm{d}^2z}{\mathrm{d}t^2} = -\mu\frac{x}{r^3} + \frac{3}{8}c_d\overline{\rho}\frac{1}{r_b} \mid \mathbf{W}_f - \mathbf{W} \mid (w_f - \frac{\mathrm{d}z}{\mathrm{d}t}) \end{cases} \tag{8}$$

where μ is the standard gravitational parameter and c_d is the aerodynamic coefficient.

4. ATMOSPHERIC MODELS SPECIFIC FOR EARTH

A reference atmosphere model determines the properties of the atmosphere depending on the altitude, latitude, longitude and time. In this paper we will use the NRLMSISE-00 and the U.S. Standard Atmosphere atmospheric models.

United States *Naval Research Laboratory Mass Spectrometer Incoherent Scatter Radar* (NRLMSISE-00) is an empirical atmospheric model that is an upgrade from the MSISE-90 model (Picone *et al.*, 2002). The input of the NRLMSISE-00 atmosphere model is: date, altitude, longitude, latitude, f107A (81 day average of F10.7 flux (centered on the given day of year)), f107 (daily F10.7 flux for previous day) and magnetic index (daily). The output of the NRLMSISE-00 atmosphere model is: He number density (cm^{-3}), O number density (cm^{-3}), N_2 number density (cm^{-3}), O_2 number density (cm^{-3}), Ar number density (cm^{-3}), total mass density (g cm^{-3}), H number density (cm^{-3}), N number density (cm^{-3}), anomalous oxygen number density (cm^{-3}), exospheric temperature and temperature at altitude (K).

In Figure 1 we represented the variation of the density of the atmosphere with respect to the altitude for the NRLMSISE-00 atmosphere model.

The 1976 U.S. Standard Atmosphere determines the values of pressure, temperature and density depending on the altitude (AIAA, 2010). The model was constructed based on temperature measurements by radiosondes, rocketsondes, rockets and satellites (AIAA, 2010) and is reliable for altitudes below 86 km.

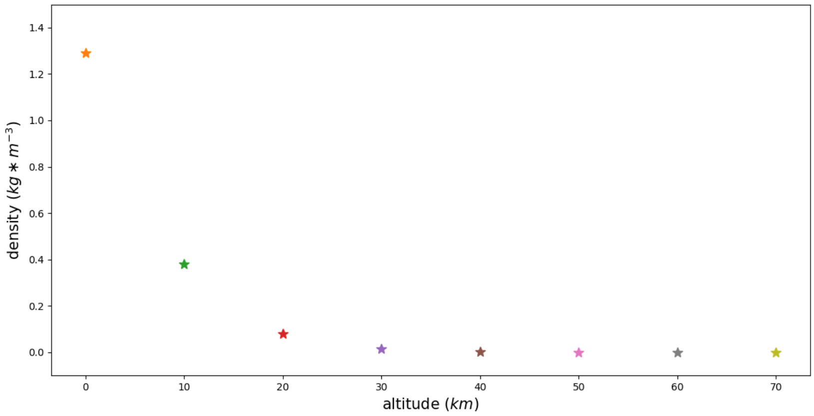

Fig. 1 – Variation of density for the NRLMSISE-00 model.

5. RESULTS

We created a Python code for implementing the algorithm described in the previous section.

The novelty of our model and of the algorithm is that it considers ellipsoidal shape of the Earth, the Coriolis effect and the centrifugal force. Each iteration during the dark-flight evaluated the meteoroid position in ECEF coordinates and the strewn field was determined in the assumption of the ellipsoidal shape of the Earth.

Several runs using various input parameters for simulated meteoroids produce a variety of results. Hereinafter we present one of these runs.

We took as input parameters: the equatorial radius of the Earth (6,378,137 m), the polar radius of the Earth (6,356,752 m), $\Omega = \frac{2\pi}{86{,}164.09}$. We consider the free fall of a spherical body with the drag coefficient of $c_d = 0.47$, the mass of 0.1 kilograms and the density of 2,000 $\frac{\mathrm{kg}}{\mathrm{m}^3}$. At the end of the ablation phenomenon the object has the speed of 2,200 $\frac{\mathrm{m}}{\mathrm{s}}$ and enters at an angle of $45°$.

In our example, the dark-flight starts at a height of 30,000 m, the longitude of $26°$ and the latitude of $44°$ (in Earth-Centered, Earth Fixed ECEF coordinates $X = 4{,}079{,}450.086$m, $Y = 1{,}989{,}680.750$m and $Z = 4{,}508{,}561.612$m).

The input data of our algorithm is in agreement with data from the literature. In order for a meteoroid to produce meteorites the dark-flight trajectory has to begin in the height interval 40km-20km (Moilanen, Gritsevich, and Lyytinen, 2021).

In Kabakchiev *et al.* (2018) the authors mention the typical velocity of a meteoroid entering the Earth's atmosphere to be $20\frac{\mathrm{km}}{\mathrm{s}}$. The speed of the meteoroid is in agreement with the data from the literature (Gritsevich, 2009). The meteoroid decelerates during the luminous phenomenon and when the dark-flight trajectory begins the object reaches the terminal velocity of a few $\frac{\mathrm{km}}{\mathrm{s}}$ (Spurný, Borovička, and Shrbený, 2020; Olech *et al.*, 2017). In Moilanen, Gritsevich, and Lyytinen (2021) the authors mention that in order to produce strewn fields similar to the known strewn fields the variations of the wind speed have to be close to $\pm 20\%$. In Kabakchiev *et al.* (2018) the authors also mention that the entry angle of a meteoroid is about $45°$-$46°$. The input values of our code are in agreement with the data from the MOROI and FRIPON networks.

The u_f and v_f wind components are affected by errors and their variation must be evaluated in a stochastic context. To account for this, we generated 21 random samples from a normal (Gaussian) distribution for the u_f and v_f components of the wind velocity. The general form of the probability density function is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

where μ is the mean and σ is the standard deviation. In our example we took $\mu = 20\frac{\mathrm{m}}{\mathrm{s}}$ and $\sigma = 5\ \frac{\mathrm{m}}{\mathrm{s}}$.

We calculated the value of the air density using the U.S. Standard Atmosphere.

Figure 2 represents the variation of height with respect to the geodetic distance between the initial point of the trajectory and the current point of the trajectory.

Figure 3 represents the variation of height with respect to the time that passes since the dark-flight phenomenon began.

Sub-figure a) from Figures 2 and 3 contains the simulations corresponding to u_f generated with a Gaussian distribution and $v_f = 0$. In blue is represented the value $u_f = 19.99\frac{m}{s}$ and in grey are plotted the first random generated and the last generated values ($u_f = 13.63\ \frac{\mathrm{m}}{\mathrm{s}}$ and $u_f = 29.81\ \frac{\mathrm{m}}{\mathrm{s}}$).

Sub-figure b) from Figures 2 and 3 contains the simulations corresponding to $u_f = 0$ and v_f is generated with a Gaussian distribution. In blue is represented the value $v_f = 20.19\ \frac{\mathrm{m}}{\mathrm{s}}$ and in grey are plotted the first random generated and the last generated values ($v_f = 10.20\ \frac{\mathrm{m}}{\mathrm{s}}$ and $v_f = 33.13\ \frac{\mathrm{m}}{\mathrm{s}}$).

Sub-figure c) from Figures 2 and 3 contains the simulations when both u_f and v_f are generated with a Gaussian distribution. In blue is represented the value $u_f = 19.57\ \frac{\mathrm{m}}{\mathrm{s}}$, $v_f = 19.78\ \frac{\mathrm{m}}{\mathrm{s}}$ and in grey are plotted the first random generated and the last generated values ($u_f = 12.63\ \frac{\mathrm{m}}{\mathrm{s}}$, $v_f = 13.27\ \frac{\mathrm{m}}{\mathrm{s}}$ and $u_f = 25.89\ \frac{\mathrm{m}}{\mathrm{s}}$, $v_f = 26.10\frac{\mathrm{m}}{\mathrm{s}}$).

Figure 4, Figure 5, and Figure 6 present the coordinates of the points where the meteorite can be found on the surface of the Earth. The background in figures 4, 5 and 6 was downloaded from OpenStreetMap †.

The Monte Carlo simulation has been used in other works (Moilanen, Gritsevich, and Lyytinen, 2021) due to the fact that the exact values of the atmosphere

† https://www.openstreetmap.org/export#map=5/51.500/-0.100

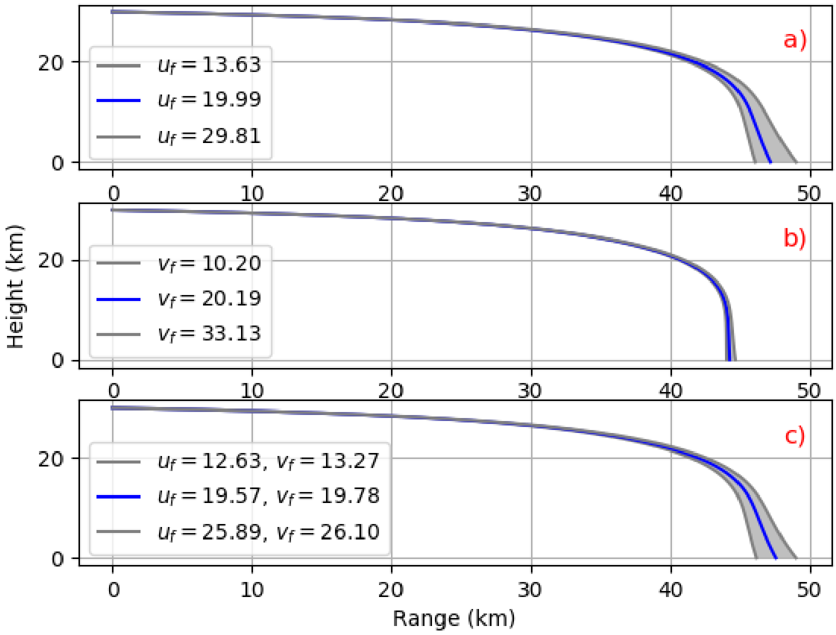

Fig. 2 – The projection on the trajectory plane at various heights. a) u_f is generated with a Gaussian distribution, $v_f = 0$. b) v_f is generated with a Gaussian distribution, $u_f = 0$. c) both u_f and v_f are generated with a Gaussian distribution.

properties and the exact values of the wind profile cannot be obtained. In Moilanen, Gritsevich, and Lyytinen (2021) the authors compare the predicted strewn field with the positions of the recovered fragments for the Košice and Neuschwanstein meteorites. Moilanen, Gritsevich, and Lyytinen (2021) mention that the shape of ellipse for the strewn field is outdated; the strewn field is more likely to have the shape of a *narrow long tie shaped area*.

In this example we obtain a diamond shape of the strewn field due to the fact that the u_f and v_f wind components were chosen independently, each having a Gaussian distribution. We mention that different shapes for the strewn field can be obtained if we make Monte Carlo simulations for more parameters (Tóth *et al.*, 2015).

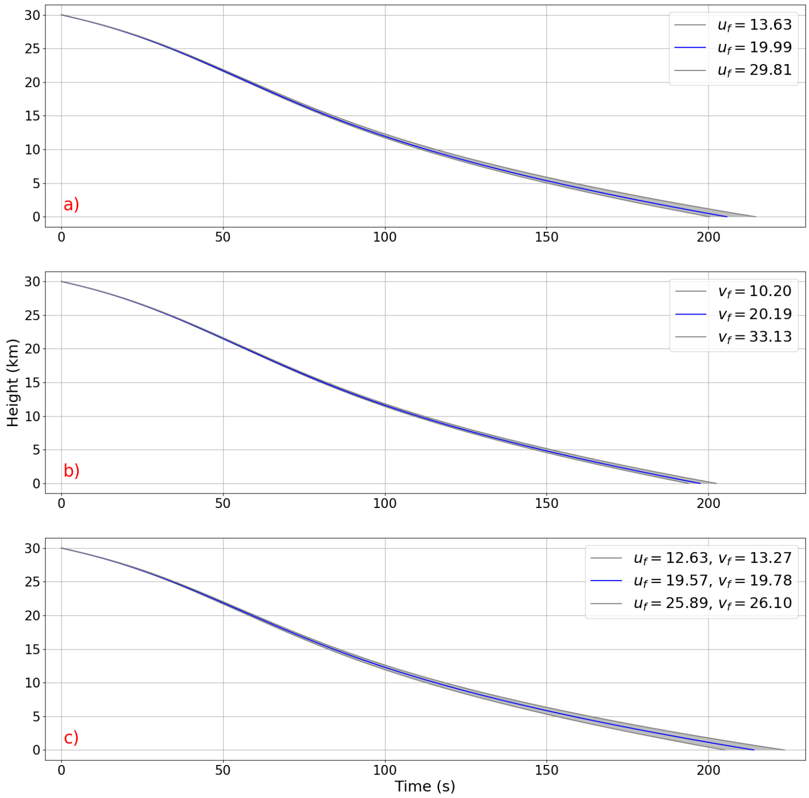

Fig. 3 – Height as a function of time. a) u_f is generated with a Gaussian distribution, $v_f = 0$. b) v_f is generated with a Gaussian distribution, $u_f = 0$. c) both u_f and v_f are generated with a Gaussian distribution.

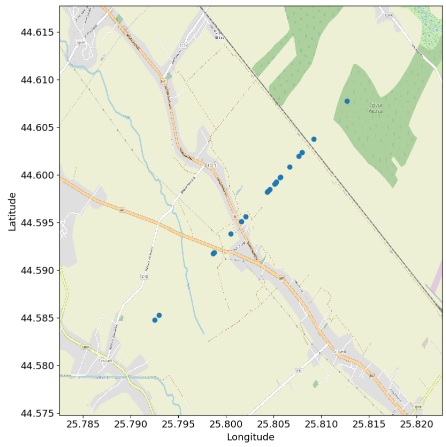

Fig. 4 – Strewn field for u_f generated with a Gaussian distribution, $v_f = 0$. The length of the strewn field is 3.011 km.

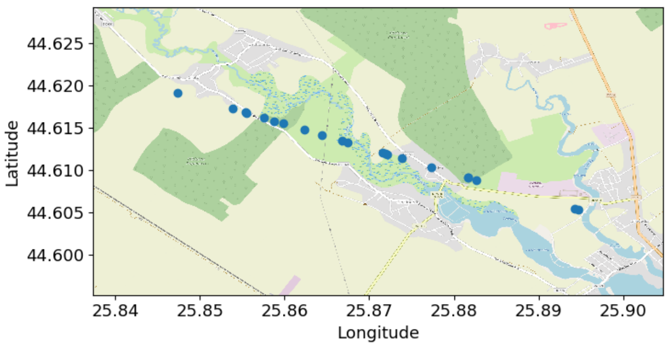

Fig. 5 – Strewn field of v_f generated for a Gaussian distribution using $u_f = 0$. The length of the strewn field is 4.05 km.

6. CONCLUSION AND FUTURE WORK

This article presents the general equations of movement for a body into a planetary atmosphere and the specificity of free fall of an object on Earth's atmosphere. The 3D model of this study takes into account Earth's ellipsoidal shape, the Coriolis and centrifugal forces, and the characteristics of the atmosphere of the Earth and its dynamics. The Monte Carlo simulations of wind components were also included in this model.

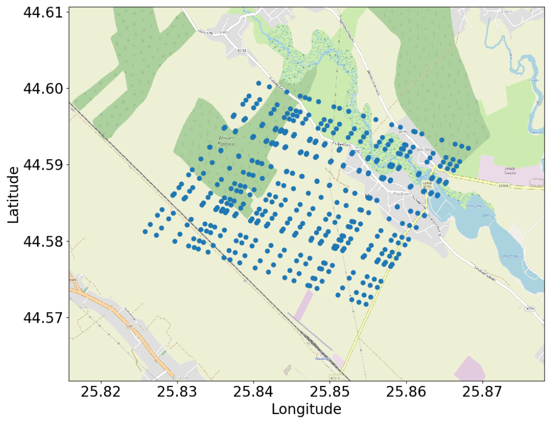

Fig. 6 – Strewn field for u_f and v_f generated with a Gaussian distribution. The dimension of the strewn field is 3.21×3.36 km.

Our model was applied to a spherical meteoroid and the results are evaluated in terms of its strewn field representing the area of probability for the object to be found on Earth surface. Our model shows the importance of local wind components in determining this path and for minimizing the ground area of fall and the thus facilitating the research for meteorites.

This mathematical model and the numerical algorithm will be applied further on data from the MOROI (Nedelcu *et al.*, 2018; Anghel *et al.*, 2019) and FRIPON (Jeanne *et al.*, 2019; Colas *et al.*, 2020) networks.

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