REISSNER-NORDSTRÖM METRIC VIOLATES WEAK EQUIVALENCE PRINCIPLE BY ROTATION

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Abstract. For circular motion, the validity of principle of weak equivalence−an ingredient of Einstein equivalence principle is investigated in the vicinity of Reissner-Nordström black hole. To accomplish it, rotational transformations with uniform angular velocity are chosen, both the *Galileo type* as well as the *Lorentz type*. Then the stationary Reissner-Nordström metric is transformed into rotational according to the transformations. All fiducial observers (FidOs) described by those rotational metrics should consequently measure similar outcome in a physical experiment at the distance of black hole's photon sphere radius as they all experience alike acceleration there−an indication of equivalence of the observers' reference frames. But the study finds anomaly for both of the transformation types, thus a violation of the weak principle of equivalence.

Key words: weak equivalence principle – Reissner-Nordström solution – photon sphere – Sagnac effect – NTO reference frame.

1. INTRODUCTION

A striking breakthrough in human consciousness regarding the notion of space and time based on Newtonian philosophy came on the stage when in 1905, Einstein said that space and time are not separate entities, rather they simultaneously manifest a single quantity as space-time sheet. Gravitation is the mere consequence of this space-time. A 1907 *happiest thought* led him to introduce *hypothesis of complete physical equivalence* between a gravitational field and an accelerated reference frame, and using it as a *Socratic* tool to ultimately construct the relativistic theory of gravitation. In its simplest form, the equivalence principle says that a gravitational field is locally * equivalent to an accelerated frame. Precisely, an observer

*The spatial characteristic ℓ of the local inertial frame is much smaller than the typical length-scale L of the gravitational field, $R_{\alpha\beta\gamma\delta} \sim 1/L^2$. Hence, in a region $\ell << L$, the field is almost homogenous, and be 'cancelled' by inertial force.

falling freely does not experience the gravity effects except through the tidal forces, *i. e.*, the curvature components (Susskind and Lindesay, 2004). This is coined as the Weak form of Equivalence Principle (WEP). A direct aftermath of the WEP is the so-called free fall universality, came from Galileo's historical *gedanken* experiment, which states that the world line of a freely falling non-gravitationally binding body [†] immersed in a gravitational field is determined, independently of its inner composition and structure, only by the surrounding geometry (Misner *et al.*, 1973).

In the course of development of general relativity, Einstein formulated even a more strict principle of equivalence compared to the former, named after him, which is the combination of the followings (Will, 2006)

- WEP is valid.
- Local Lorentz invariance (the outcome of any local non-gravitational experiment [‡] is independent of the velocity and orientation of the freely-falling reference frame in which it is performed) is valid.
- Local position invariance (the outcome of such an experiment is independent of when and where in the universe it is performed) is also valid.

The statement that 'the gravity-effects are replaceable by curved 4-dimensional spacetime' gets validation from the above Einstein equivalence principle, which contains the weak one. Hierarchically, a violation of the free-fall universality would invalidate WEP, hence the Einstein equivalence principle, and thus placing a limit on the validity of relativity theory. This instigates the universality of free fall to be tested by experimental as well as theoretical machinery. The quantification of the versatile free-fall measurement is the famous Eötvös parameter, $\eta = \Delta a/a$. The finding of a value $\eta \neq 0$ would obviously indicate the WEP-violation through a disagreement with the ubiquitous free fall. More and more stringent bounds on η had been imposed throughout the last decades and is being imposed still to date, through several types of experiments such as torsion balance, lunar laser ranging, spaced based, gyro-physical etc., (Su *et al.*, 1994; Nobili *et al.*, 2008; Fray *et al.*, 2004).

Quantum mechanics, another conquering physics theory shows reluctance to merge with the general theory of relativity, where the first one fits well in the realm of micro-cosmos and the second one in macro. The crucial cause seems that where

[†] Electrically neutral particle which has negligible gravitational binding energy, negligible angular momentum, and which is small enough that the gravitational field within its volume is homogenous.

[‡] Process involving particle and continuum mechanics, thermodynamics, electromagnetism etc., provided that they do not affect the previously existing background gravitational field, nor create even a non-negligible one in case there is none.

quantum mechanics governed by the uncertainty principle is purely non-local, the relativity theory is directly opposite. It has been suspected that a theory of quantum gravity may not be possible (Robinowitz, 2006), for quantum version of the WEP [§] is clearly violated. The free fall of test particles in a uniform gravitational field in the case of quantum states with and without a classical analogue has been re-examined by Viola and Onofrio (1997), who have found the violation. Another quantum mechanical approach of the WEP violation for a quantum particle using a model quantum clock has been presented (Peres, 1980).

Variability framework of electromagnetic fine-structure constant α (Bekenstein, 1982) in which the exponent of a scalar field plays the role of permittivity and inverse permeability, has long been connected with the possibility of WEP violation (Uzan, 2003) coming from the classical Coulomb energy contribution to the particle masses. Strict upper bounds have been set from Eötvös experiment on the Bekenstein parameter when the electrostatic field does generate external dilaton field (Mosquera *et al.*, 2008) or not (Kraiselburd and Vucetich, 2011). A quantum correction to the Brans-Dicke theory due to interaction among matter fields results in violation of the WEP. In the one-loop correction, portion proportional to α sees a finite extra term giving the difference between what the tensor gravitational field feels and what the scalar field feels (Fujii, 1994).

According to the violation of equivalence principle mechanism (Gasperini, 1988) in which neutrino does not necessarily have non-zero mass, neutrino oscillations occur if gravity has not universal coupling to leptonic flavors. The sensitivity of oscillation to the violation emerges from the fact that the flavor states of neutrinos are a coherent superposition of mass eigenstates. Violation effectively changes the mass-squared differences by adding a term proportional to the neutrino energy-squared. High energy atmospheric neutrino data collected by IceCube (IC-40 and IC-79) put bounds on violation parameter (Esmaili *et al.*, 2014). However, results from Liquid Scintillator Neutrino Detector together with laboratory experiments (Angelina *et al.*, 1986; Zacek *et al.*, 1986) show that massless or degenerate mass neutrinos with flavor non-diagonal gravitational couplings rule out violation mechanism (Mann and Sarkar, 1996).

The interest to check the validity of weak equivalence principle in the space-time regions described by the solutions of Einstein field equations is new. A recent

[§] 'The results of experiments in an external potential comprising just a sufficiently weak, homogeneous gravitational field, as determined by the wave function, are independent of mass of the system' (Holland, 1995).

study (Jensen, 2007) has shown that the weak principle breaks down in Schwarzschild geometry, where the rotation was *Galileo-type* (absolute time), somewhat analogous to the Galileo transformation in translational motion of the theory of relativity. In this work, the previous study is extended by the inclusion of a charge parameter with the Schwarzschild metric. Plus, the *Lorentz type* rotational transformation is considered. In Section 2, the overall mathematical formulations have been presented, the *Galileo type* in section 2.1 and the *Lorentz type* in section 2.2. In both sections, rotational Reissner-Nordström metric components according to the choice of the rotational transformations have been derived. Then the proper time for a FidO and for the photon relatively rotating uniformly with respect to the FidO have been calculated. After that, the speed of the light particle from the viewpoint of the FidO has been measured. It is evident that not all the observers measure similar result. Section 3.1 shows the equivalence of FidO frames in the gravitational field of Reissner-Nordström black hole, section 3.2 claims the violation of WEP and section 3.3 discusses the causes for the violation of the principle. Finally, some concluding remarks have been presented in Section 4.

2. CALCULATION OF PHOTON PROPAGATION SPEED

Reissner-Nordström metric, the relativistic stationary, spherically symmetric, charged, and asymptotically flat solution to the coupled Einstein-Maxwell equations in spherical polar coordinate system (t, r, θ, φ) is

$$\begin{aligned} \mathrm{d}s^2 &= g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \\ &= -f(r)c^2\mathrm{d}t^2 + \frac{1}{f(r)}\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2, \end{aligned} \tag{1}$$

in which

$$f(r) = 1 - \frac{2M}{c^2 r} + \frac{Q^2}{c^4 r^2},$$

where M is the central gravitational mass, and Q is the net charge comprised of electric and magnetic charges through the relationship $Q^2 = Q_e^2 + Q_m^2$. The metric signature is $(-+++)$, widely used in general relativity. Here the universal gravitational constant G is measured in Planck unit, *i. e.*, $G = 1$, but not the light velocity c. The non-null and stationary Reissner-Nordström line elements $g_{\mu\nu}$ are

$$g_{tt} = -c^2 + \frac{2M}{r} - \frac{Q^2}{c^2 r^2},$$

$$g_{rr} = \frac{1}{1 - \frac{2M}{c^2 r} + \frac{Q^2}{c^4 r^2}},$$

$$g_{\theta\theta} = r^2,$$
$$g_{\varphi\varphi} = r^2\sin^2\theta.$$

2.1. *GALILEO TYPE* ROTATIONAL FORMALISM

2.1.1. Rotational metric components

A widely accepted (Adler, 1975; Grøn, 1975; Pellegrini and Swift, 1995; Klauber, 1998) rotational transformation with constant angular velocity Ω about the equatorial plane is

$$\left.\begin{aligned} \mathrm{d}t &= \mathrm{d}\bar{t}, \\ \mathrm{d}r &= \mathrm{d}\bar{r}, \\ \mathrm{d}\theta &= \mathrm{d}\bar{\theta}, \\ \mathrm{d}\varphi &= \mathrm{d}\bar{\varphi} - \Omega\mathrm{d}\bar{t}. \end{aligned}\right\} \tag{2}$$

It is obvious that transformation (2) is diffeomorphic invariant. Thence the previously noted non-rotated and the later-derived rotated metric components are subjects to remain on the same manifold. Assuming $\mathrm{d}s$ as invariant, the metric for the rotating coordinate grid is found as

$$\mathrm{d}s^2 = -c^2\left[f(\bar{r}) - \frac{\Omega^2\bar{r}^2\sin^2\bar{\theta}}{c^2}\right]\mathrm{d}\bar{t}^2 + \frac{1}{f(\bar{r})}\mathrm{d}\bar{r}^2 + \bar{r}^2\mathrm{d}\bar{\theta}^2 - 2\Omega\bar{r}^2\sin^2\bar{\theta}\mathrm{d}\bar{t}\mathrm{d}\bar{\varphi} + \bar{r}^2\sin^2\bar{\theta}\mathrm{d}\bar{\varphi}^2. \tag{3}$$

Metric (3) is the so-called Langevin metric (Langevin, 1921). Its non-diagonality indicates that the bar-frame is Non-Time Orthogonal (NTO). The transformed metric components

$$\bar{g}_{tt} = -c^2 + \frac{2M}{\bar{r}} - \frac{Q^2}{c^2\bar{r}^2} + \Omega^2\bar{r}^2\sin^2\bar{\theta},$$
$$\bar{g}_{rr} = \frac{1}{1 - \frac{2M}{c^2\bar{r}} + \frac{Q^2}{c^4\bar{r}^2}},$$
$$\bar{g}_{\theta\theta} = \bar{r}^2,$$
$$\bar{g}_{\varphi\varphi} = \bar{r}^2\sin^2\bar{\theta},$$
$$\bar{g}_{t\varphi} = \bar{g}_{\varphi t} = -\Omega\bar{r}^2\sin^2\bar{\theta},$$

are for a FidO rotating about the singularity of the Reissner-Nordström black hole.

2.1.2. Radius of the photon sphere

Although tremendous efforts have been pursued on the topic of the photon sphere of Reissner-Nordström black hole (Chakraborty and Chakraborty, 2011; Khoo and Ong, 2016), the radius of photon sphere is calculated here from the equation of the effective potential. V, the effective potential, experienced by a massless particle satisfies the equation

$$V(r) = \frac{J^2}{r^2} f(r),$$

where J denotes the particle's angular momentum, and $f(r)$ is the lapsus function. The existence of a photon orbit corresponds to a stationary point on the potential, *i.e.* a root of the equation

$$V'(r) = \frac{J^2}{r^2}\left[f'(r) - \frac{2}{r} f(r)\right] = 0,$$

where $' \equiv \frac{\mathrm{d}}{\mathrm{d}r}$. This gives

$$V'(r) = \frac{J^2}{r^2}\left[\frac{6M}{c^2 r^2} - \frac{4Q^2}{c^4 r^3} - \frac{2}{r}\right] = 0.$$

Then the solutions for r are,

$$r_{\pm} = \frac{3M \pm \sqrt{9M^2 - 8Q^2}}{2c^2}.$$

The root

$$r_{-} = \frac{3M - \sqrt{9M^2 - 8Q^2}}{2c^2},$$

cannot be the radius of photon sphere since it lies between the event horizon and the Cauchy horizon of the Reissner-Nordström black hole. In that region, $\partial/\partial r$ is time-like (Claudel *et al.*, 2001).

2.1.3. Proper time for FidO and the photon

As for a time-like world line on which a subluminal observer 'actually' moves, the space-time interval $\mathrm{d}s^2$ is negative, so the proper time τ for a FidO satisfies the relation

$$c^2 \mathrm{d}\tau^2 = -\mathrm{d}s^2 = -\bar{g}_{\mu\nu} \mathrm{d}\bar{x}^{\mu} \mathrm{d}\bar{x}^{\nu}.$$

This immediately leads to the proper time for FidO measured in a time interval $0 \leq \bar{t} \leq T$,

$$\begin{aligned} \tau &= \frac{1}{c}\int_0^T \sqrt{-\bar{g}_{\mu\nu}\frac{\mathrm{d}\bar{x}^{\mu}}{\mathrm{d}\lambda}\frac{\mathrm{d}\bar{x}^{\nu}}{\mathrm{d}\lambda}}\, \mathrm{d}\lambda \\ &= \frac{1}{c}\int_0^T \sqrt{-\bar{g}_{tt}}\, \mathrm{d}\bar{t}, \end{aligned} \tag{4}$$

where λ is a time-like parameter. The negative sign for a time-like world line assures the positivity of space-time interval. A FidO rotating at the distance of photon sphere radius of Reissner-Nordström black hole, $\bar{r} = R_{\text{ph}} = \frac{3M+\sqrt{9M^2-8Q^2}}{2c^2}$ in the equatorial plane, $\bar{\theta} = \pi/2$ finds its proper time from the expression (4) as

$$\tau = \frac{1}{c}\sqrt{\frac{2\left(3M^2-2Q^2+M\sqrt{9M^2-8Q^2}\right)}{\left(3M+\sqrt{9M^2-8Q^2}\right)^2} - \Omega^2\frac{9M^2-4Q^2+3M\sqrt{9M^2-8Q^2}}{2c^4}}\,T. \quad (5)$$

From the subluminality condition of the rotating observer, one gets bound on Ω from (5) as

$$\Omega \leq \left|\frac{2\sqrt{2}c^2\sqrt{3M^2-2Q^2+M\sqrt{9M^2-8Q^2}}}{\left(3M+\sqrt{9M^2-8Q^2}\right)^2}\right|.$$

Consider

$$K(\Omega) = \sqrt{\frac{2\left(3M^2-2Q^2+M\sqrt{9M^2-8Q^2}\right)}{\left(3M+\sqrt{9M^2-8Q^2}\right)^2} - \Omega^2\frac{9M^2-4Q^2+3M\sqrt{9M^2-8Q^2}}{2c^4}},$$

and assume that a particle moves at R_{ph} in the equatorial plane with constant angular velocity ω relative to the FidO as viewed from infinity. Then the proper time found for the particle is $\frac{1}{c}K(\Omega+\omega)T$. If this particle is massless photon, then it follows null geodesic to vanish its proper time, *i. e.*,

$$K(\Omega+\omega) = 0,$$

from which photon's angular velocity ω is found as

$$\omega = -\Omega \pm \frac{2\sqrt{2}c^2\sqrt{3M^2-2Q^2+M\sqrt{9M^2-8Q^2}}}{\left(3M+\sqrt{9M^2-8Q^2}\right)^2}. \quad (6)$$

2.1.4. Photon speed relative to FidO

The angular velocity of photon observed from infinity is the measure of expression (6). However, the velocity for particle tracing null geodesic, collaborated in the FidO frame is

$$\omega' = \omega\frac{\mathrm{d}T}{\mathrm{d}\tau}.$$

Accordingly the speed of light particle, c' ascertained locally by FidO is

$$c' = \omega' R_{\text{ph}},$$

which implies that

$$c' = \frac{c\left(3M+\sqrt{9M^2-8Q^2}\right)}{2K(\Omega)}\left[-\Omega \pm \frac{2\sqrt{2}c^2\sqrt{3M^2-2Q^2+M\sqrt{9M^2-8Q^2}}}{\left(3M+\sqrt{9M^2-8Q^2}\right)^2}\right], \tag{7}$$

where positive and negative signs stand for the same and the opposite-directed FidOs respectively with respect to the propagation direction of photon. It is evident from the light-velocity expression (7) that the observer with the trivial case $\Omega = 0$ observes light velocity $c' = \pm c^3$. In spite of scattering discriminately, the light particles still follow light-like geodesics according to the perspective of any FidO, *i.e.* the geodesic equations

$$\bar{g}_{\mu\nu}\frac{d\bar{x}^\mu}{d\lambda}\frac{d\bar{x}^\nu}{d\lambda} = 0,$$

and

$$\frac{d^2\bar{x}^k}{d\lambda^2} + \Gamma^k_{ij}\frac{d\bar{x}^i}{d\lambda}\frac{d\bar{x}^j}{d\lambda} = 0,$$

are satisfied, where $\Gamma^k_{ij} = \frac{1}{2}\bar{g}^{kr}\left(\partial_i\bar{g}_{jr} + \partial_j\bar{g}_{ri} - \partial_r\bar{g}_{ij}\right)$ are the Christoffel symbols of the second kind. λ is any parameter for null geodesics (except proper time).
The inverse of the rotating metric components are

$$\bar{g}^{tt} = \frac{1}{-c^2 + \frac{2M}{\bar{r}} - \frac{Q^2}{c^2\bar{r}^2}},$$

$$\bar{g}^{rr} = 1 - \frac{2M}{c^2\bar{r}} + \frac{Q^2}{c^4\bar{r}^2},$$

$$\bar{g}^{\theta\theta} = \frac{1}{\bar{r}^2},$$

$$\bar{g}^{\varphi\varphi} = \frac{1}{\bar{r}^2\sin^2\bar{\theta}} - \frac{\Omega^2}{c^2 - \frac{2M}{\bar{r}} + \frac{Q^2}{c^2\bar{r}^2}},$$

$$\bar{g}^{t\varphi} = \bar{g}^{\varphi t} = \frac{\Omega}{c^2 - \frac{2M}{\bar{r}} + \frac{Q^2}{c^2\bar{r}^2}}.$$

2.2. *LORENTZ TYPE* ROTATIONAL FORMALISM

Transformation (2) can be interpreted as an analogous Galileo counterpart of translational motion in relativity theory. Post, in his popular work (Post, 1967), added an extra factor γ to the transformations (2) noted here, which could be unity, Lorentz contraction factor or anything else. The motivation of the additional factor was to conduct higher order analysis. Now his 'transformation (11)', what is being called the *Lorentz type* in this work, will be followed in the subsequent sections.

2.3. ROTATIONAL METRIC COMPONENTS

In this case, the rotation transformation is

$$\left.\begin{aligned} \mathrm{d}t &= \gamma \mathrm{d}\bar{t}, \\ \mathrm{d}r &= \mathrm{d}\bar{r}, \\ \mathrm{d}\theta &= \mathrm{d}\bar{\theta}, \\ \mathrm{d}\varphi &= \mathrm{d}\bar{\varphi} - \gamma\Omega \mathrm{d}\bar{t}, \end{aligned}\right\} \tag{8}$$

where $\gamma = \frac{1}{\sqrt{1-(\Omega\bar{r}/c)^2}}$. Transformation (8) reduces to transformation (2) in case of $\gamma = 1$. Thence the rotational line elements are

$$\bar{g}_{tt} = -c^2 + \frac{2M}{\bar{r}} - \frac{Q^2}{c^2\bar{r}^2} + \gamma^2\Omega^2\bar{r}^2\sin^2\bar{\theta},$$

$$\bar{g}_{rr} = \frac{1}{1 - \frac{2M}{c^2\bar{r}} + \frac{Q^2}{c^4\bar{r}^2}},$$

$$\bar{g}_{\theta\theta} = \bar{r}^2,$$

$$\bar{g}_{\varphi\varphi} = \bar{r}^2\sin^2\bar{\theta},$$

$$\bar{g}_{t\varphi} = \bar{g}_{\varphi t} = -\gamma\Omega\bar{r}^2\sin^2\bar{\theta}.$$

2.3.1. Proper time for FidO and the photon

The proper time for a rotating FidO at the distance of photon sphere in the equatorial plane is

$$\tau = \frac{1}{c}K(\Omega)T,$$

where

$$K(\Omega) = \sqrt{\frac{2\left(3M^2 - 2Q^2 + M\sqrt{9M^2-8Q^2}\right)}{\left(3M+\sqrt{9M^2-8Q^2}\right)^2} - \frac{\Omega^2c^2\left(3M+\sqrt{9M^2-8Q^2}\right)^2}{4c^6 - \Omega^2\left(3M+\sqrt{9M^2-8Q^2}\right)^2}}.$$

From the vanishing proper time $K(\Omega+\omega)$ of massless photon with angular frequency ω with respect to FidO, one gets,

$$\omega = -\Omega \pm \frac{2\sqrt{2}c^3\sqrt{3M^2-2Q^2+M\sqrt{9M^2-8Q^2}}}{\left(3M+\sqrt{9M^2-8Q^2}\right)\sqrt{c^2\left(3M+\sqrt{9M^2-8Q^2}\right)^2 + 6M^2 - 4Q^2 + 2M\sqrt{9M^2-8Q^2}}}.$$

2.3.2. Photon speed relative to FidO

A FidO, in a similar way of analysis in section 2.1.4, measure the speed of photon as

$$c' = \frac{c\left(3M+\sqrt{9M^2-8Q^2}\right)}{2K(\Omega)}$$
$$\left[-\Omega \pm \frac{2\sqrt{2}c^3\sqrt{3M^2-2Q^2+M\sqrt{9M^2-8Q^2}}}{\left(3M+\sqrt{9M^2-8Q^2}\right)\sqrt{c^2\left(3M+\sqrt{9M^2-8Q^2}\right)^2+6M^2-4Q^2+2M\sqrt{9M^2-8Q^2}}}\right]. \tag{9}$$

For *Lorentz type* rotational motion, a FidO with $\Omega = 0$ gets the light speed c' as

$$c' = \pm\frac{c^4\left(3M+\sqrt{9M^2-8Q^2}\right)}{\sqrt{c^2\left(3M+\sqrt{9M^2-8Q^2}\right)^2+6M^2-4Q^2+2M\sqrt{9M^2-8Q^2}}}.$$

The inverse rotational components are

$$\bar{g}^{tt} = \frac{1}{-c^2+\frac{2M}{\bar{r}}-\frac{Q^2}{c^2\bar{r}^2}},$$
$$\bar{g}^{rr} = 1-\frac{2M}{c^2\bar{r}}+\frac{Q^2}{c^4\bar{r}^2},$$
$$\bar{g}^{\theta\theta} = \frac{1}{\bar{r}^2},$$
$$\bar{g}^{\varphi\varphi} = \frac{1}{\bar{r}^2\sin^2\bar{\theta}}-\frac{\gamma^2\Omega^2}{c^2-\frac{2M}{\bar{r}}+\frac{Q^2}{c^2\bar{r}^2}},$$
$$\bar{g}^{t\varphi} = \bar{g}^{\varphi t} = \frac{\gamma\Omega}{c^2-\frac{2M}{\bar{r}}+\frac{Q^2}{c^2\bar{r}^2}}.$$

3. DISCUSSION

3.1. EQUIVALENCE OF FIDO FRAMES

Black holes, probably, are the weirdest objects in the universe as they give no straightforward evidence of their existence, although they are elegant mathematical manifestations of the relativity theory. A black hole is seen (surely in fantasy !) as a space-time region, *i. e.*, what characterize a black hole is its metric, and consequently its space-time curvature. All our intuitive ken gets awfully stuck with the black hole region. Peculiarity is that a black hole space-time is causally disconnected from the rest, no events in this region can make any influence on the events outside. The events in the black hole region, nonetheless, are causally decided by past events. Photon sphere of a black hole is the most distant stable spherical region from infinity where photon is compelled to orbit around the singularity due to an intensified gravitational pull. It has been shown (Abramowicz and Lasota, 1974) that in a space-time described by the Schwarzschild metric, an object with constant circumferential velocity, confined subluminally to the spherical surface of radius $3GM/c^2$, M being the mass of the gravitational body located at the origin, experiences constant acceleration,

$$a \equiv (a_0, a_1, a_2, a_3) \equiv \left(0, \frac{-c^4}{3GM}, 0, 0\right).$$

This demonstration, having nothing to do with electromagnetic phenomena, holds equally true when the considered space-time metric is Reissner-Nordström, because both the Schwarzschild and in the limit $Q \to 0$, the Reissner-Nordström black hole have similar photon sphere radius (McBryan, 2013). Reissner-Nordström metric is the generalization of Schwarzschild metric through the inclusion of electric and magnetic charges. So it can be concluded that the reference frames of all the observers are equivalent in the vicinity of a charged static black hole's gravitational field.

3.2. VIOLATION OF WEP

The WEP says that the motion of freely-falling particles is the same in a gravitational field and at a uniformly accelerated frame immersed in a small enough regions of space-time (Carrol, 2004). The tag 'small enough' indicates that the region is an arbitrarily open subset of space-time manifold, a neighborhood under the Alexandrov topology (Beem *et al.*, 1996).

The feature presented in this work is about an experiment of determining the propagation speed of photon in the photon sphere region of a Reissner-Nordström black hole gravitational field. An important fact to consider that expressions (7) and (9) and their derivations indicate no assumption about the size of the space-time region in which the speed of light measurement has to be made. Thus the speed of light given by (7) and (9) can be measured within an arbitrarily small neighborhood of space-time. Therefore the FidO frame can be treated as local in the experiment. Because all the rotating FidOs rotate with an unchanged tangential velocity, their local frames are equivalent by the WEP, meaning that freely falling particles should behave the same in all such frames. But a contrast has been found. Although it is not under the Reissner-Nordström metric itself for which the light-speed anomaly arises but rather the metric under a rotational transformation, mathematically the focus stays on the same manifold since the transformations (2) and (8) are diffeomorphisms. A diffeomorphism preserves the manifold. Moreover, physically, since any space-time can be considered and can be rotated about them, any valid space-time must not imply a contradiction when viewed by a rotating observer, even though it may not be problematic when viewed from rest. Again, for a spherically symmetric space-time, one can analyse an issue in a specific plane, *e.g.*, the plane described by $\theta = \pi/2$. But, in spite of not being the metric (3) spherically symmetric, the analyses here have been pursued in the equatorial plane for convenience. It is not also problematic, because all the FidO frames are in the same gravitational field of the black hole. Therefore, a conclusion can be drawn that WEP does not hold in the Reissner-Nordström space-time.

3.3. CAUSES FOR THE VIOLATION OF WEP

The cause for violation of the WEP here is the NTO reference frame, supported by the Sagnac effect. In the Sagnac efect (Sagnac, 1913), two light beams, sent *cw* and *ccw* around a closed path of a rotating disk, take different time intervals to travel the path. For a circular path of radius R, the difference can be represented as $\Delta t = 2v\ell/c^2$, where $v = \omega R$ is the speed of the circular motion and $\ell = 2\pi R$ is the circuit length covered by the light beam. A similar experiment is the 'Hafele-Keating' (Hafele and Keating, 1972a,b), in which the travelling time around the earth measured by a clock travelling in the same direction as the earth rotates, a clock moving in the opposite direction, and one at rest on the surface of the earth were compared. The result was that the clock travelling in the same direction as the earth rotates showed shortest travelling time, and the one moving in the opposite direction with the same velocity relative to the surface of the earth, showed the longest travelling time. This experiment can be treated as the temporal version of the Foucault pendulum, making it possible to measure the rotation of the reference frame in which the experiment is performed.

NTO (Non-Time Orthogonal) reference frame is that in which the temporal axis is not orthogonal to at least one of the spatial axes. The presence of the non-null line element term $\mathrm{d}\bar{t}\mathrm{d}\bar{\varphi}$ in metric (3) shows that in 4-dimensional space-time, axis $\bar{t}$ (time axis) is not orthogonal to the spatial axis for the circumferential direction. Specifically, if the basis vectors for the time coordinate and the azimuthal direction are $\bar{\mathbf{e}}_{\mathbf{t}}$ and $\bar{\mathbf{e}}_{\varphi}$ respectively, then $\bar{\mathbf{e}}_{\mathbf{t}} \cdot \bar{\mathbf{e}}_{\varphi} \neq 0$. It has been shown that NTO frames exhibit non-invariance and non-isotropy in the local, physical speed of light, to a degree depending on the non-time-orthogonality (Klauber, 2000).

4. CONCLUSION

It is a well known fact that light particles show speed anisotropy in a rotating frame. In this study, this fact has been applied to probe the validity of weak equivalence principle in the space-time described by the Reissner-Nordström metric. The end result found is that the principle no longer remains valid for the circular motion at the distance of photon sphere radius of Reissner-Nordström black hole.

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Received on 13 June 2017