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EFFECTS OF VARIABLE MASS, DISK–LIKE STRUCTURE, AND RADIATION PRESSURE ON THE DYNAMICS OF CIRCULAR RESTRICTED THREE–BODY PROBLEM

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Abstract. In this paper, we intend to investigate the dynamics of the Circular Restricted Three-Body Problem. Here we assumed the primaries as the source of radiation and have variable mass. The gravitational perturbation from disk-like structure are also considered in this study. There exist five equilibrium points in this system. By considering the combined effect of disk–like structure and the mass transfer, we found that the classical collinear equilibrium points depart from the x–axis. We called these equilibrium points as quasi–collinear equilibrium points. Meanwhile, this combined effect also breaks the symmetry of triangular equilibrium point positions. We noted that the quasi–equilibrium points are unstable whereas the triangular equilibrium points are stable if the mass ratio μ is smaller than critical mass μ_c. Besides the mass ratio, the stability of triangular equilibrium points depend on time.

Key words: restricted three–body problem, variable mass, disk–like structure, radiation pressure.

1. INTRODUCTION

Circular Restricted Three–Body Problem (CRTBP) consists of the movement of the third body with respect to the two primaries. The primaries move in a circular orbit and the third body is influenced by but not influences the primaries. In the classical case, the primaries and the third body are assumed as the point mass (see e.g. Roy, 2004; Murray and Dermott, 2000). There exist five equilibrium points which are divided into two categories named collinear equilibrium points L_1, L_2, and L_3 and triangular equilibrium points L_4 and L_5. The collinear equilibrium points are always unstable while the triangular equilibrium points are stable if the mass ratio $\mu < \mu_c = 0.038520896504551$.

The complexity of nature has made the CRTBP not suitable for some cases. Therefore, some authors have tried to develop the CRTBP by incorporating various effects. For instance, Radzievskii (1950) and Chernikov (1970) have considered the effect of photogravitation in the CRTBP for mimicking the stellar objects. More recently, the influence of disk-like structure has been incorporated in the CRTBP (see e.g. Chermnykh, 1987; Jiang and Yeh, 2004). There are several studies that combined various additional effects in CRTBP. For instance, Singh and Taura (2013) has studied CRTBP by assuming both primaries are radiating and oblate bodies, together with the effect of disk-like structure. Nurul Huda *et al.* (2023) combined the effect of photogravitational and disk-like structure, with addition of oblateness and finite-straight segment for the primaries, to study the stability of equilibrium points in CRTBP.

Several close binary star systems have been discovered (Price-Whelan *et al.*, 2020; Tutukov and Cherepashchuk, 2020). It has already studied that some of them have planets that have mass much less compared to the binary (Gong and Ji, 2018; Thebault and Haghighipour, 2015). Meanwhile, previous studies also suggest that there is also the possibility that an asteroid belt-like structure also exists in the binary system (Bancelin *et al.*, 2015; Jennings, Cordes, and Chatterjee, 2020). In certain cases, the transfer of mass between binary stars is unavoidable (Qian *et al.*, 2020). However, accurately predicting how mass moves between close–orbiting stars is still a major challenge. In the case of CRTBP, the transfer mass between stars in a binary system can be modelled by the variability of mass of each primary. The study of variable mass in the restricted three-body problem was done starting from the 1930s by Orlov (1939). More recently, Luk'yanov (2005) studied the CRTBP system in which the primaries have variable masses but the sum of their masses remains constant. Singh and Leke (2012) consider the variation of mass of primaries in accordance with the combined Meshcherskii law. Several studies also consider the variable mass of the third body (see e.g. Albidah and Ansari, 2023; Suraj *et al.*, 2021; Abouelmagd and Mostafa, 2015).

In this study, we investigate the possible movement of the infinitesimal mass in the close binary star system. We used a framework of CRTBP where the binaries are primaries. We assumed that the stars emit radiation and there is a mass transfer between primaries. We also considered a disk-like structure surrounding this three-body system, mimicking the Kuiper or asteroid belt structure.

This paper is outlined as follows. In Section 2, we give a detail about the equation of motion of the system. The detail about the equilibrium points is given in Section 3. Section 4 describes the stability of the system. Finally, the conclusion is given in Section 5. Here we used *Mathematica* software to conduct a numerical calculation or algebraic manipulation.

2. EQUATION OF MOTION

Let the mass of the first and second primaries be m_1 and m_2, respectively. The mass ratio between primaries is represented by $\mu = m_2/(m_1+m_2)$. Here we defined the mass ratio as $0 < \mu < 1$ following Luk'yanov (2005). The mass of the primaries is represented by $1-\mu$ and μ. To simplify the problem, we considered the system in a two dimensional rotational coordinate Oxy with the primaries always laying on x–axis. The position of the third body is represented by $(x,\ y)$. The origin of the coordinate system is located in the position of m_1. We take the distance between primaries as the unit of length and the unit of time is chosen in such a way so that the gravitational constant is unity.

The radiation force (F_p) has an opposite direction with respect to the gravitational force (F_g). In order to consider the radiation pressure in the CRTBP, we defined the mass reduction factor $q = 1-(F_p/F_g)$, where $0 < 1-q \ll 1$. Meanwhile, the disk–like structure effect can be modelled by following Miyamoto and Nagai (1975). The potential of disk–like structure for planar version is given as $V(x,y) = M_b/\sqrt{r^2+T^2}$, where M_b is the total mass of the disk–like structure and $r^2 = x^2+y^2$ is the radial distance of the infinitesimal mass. The mass parameter of the disk–like structure is M_b and is assumed to be small compared to the total mass of the primaries ($M_b \ll 1$). Here $T = a+b$ means the density profile of the dust belt, where a and b are the flatness and core parameters of the disc respectively. Figure 1 shows a graphic representation of the system. It has a center of inertia (A) as its center. This center is a static point. There is also center of mass (D) that moves around point A. It has to be noted that in our case the distance between A and D is so small.

For simplicity, we shall consider conservative linear mass transfer law between the primaries,

$$\mu(t) = \frac{m_2(t)}{m_1(t)+m_2(t)} = kt. \tag{1}$$

Here t means time and $0 < t < \frac{1}{k}$, where k is the rate of transfer. It has to be noted that the sum of mass $m_1(t)$ and $m_2(t)$ is constant. We assume that k is much slower compared to the orbital period of the primaries, *i.e.* $k \ll \frac{1}{n}$, where n is the mean motion of the two body system,

$$n^2 = 1+\frac{2M_b r_c}{(r_c^2+T^2)^{3/2}}. \tag{2}$$

The reference radius of the disk–like structure is given by $r_c^2 = 1-\mu+\mu^2$ as in Singh and Taura (2013). Assuming that the transfer mass between primaries is very slow and that the dominant order in the expansion is the first order, we have

$$\mu(t) \approx \mu(t_0)+\dot{\mu}(t_0)(t-t_0), \tag{3}$$

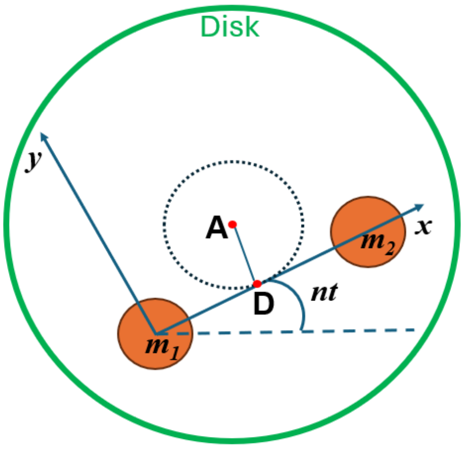


Fig. 1 – Schematic diagram of the system in this study.

where we could define $\dot{\mu}(t_0) \equiv k$ and $\mu(t_0) \equiv \mu_0$, so that

$$\mu(t) = \mu_0 + k(t - t_0). \tag{4}$$

Note that for $\mu_0 = kt_0$, equation 4 reverts back to equation 1. In the case of $\mu_0 = kt_0$, the domain for t is $\left(t_0 - \frac{\mu_0}{k}\right) < t < \left(t_0 + \frac{1-\mu_0}{k}\right)$.

The equation of motion of the system is given as follows

$$\begin{aligned} \ddot{x} - 2n\dot{y} &= W_x, \\ \ddot{y} + 2n\dot{x} &= W_y, \end{aligned} \tag{5}$$

where W_x and W_y mean the partial derivative of W with respect to x and y respectively. The pseudo potential is given by

$$W = \frac{1}{2}n^2(x^2 + y^2) - \mu n^2 x + \frac{(1-\mu)q_1}{r_1} + \frac{\mu q_2}{r_2} + \frac{M_b}{(\mathcal{R}^2 + T^2)^{1/2}}. \tag{6}$$

The first derivatives of the pseudo potential with respect to the third body position is as follows

$$\begin{aligned} W_x &= n^2 x - \mu n^2 - \frac{(1-\mu)q_1 x}{r_1^3} - \frac{\mu q_2 (x-1)}{r_2^3} - \frac{M_b(x-\mu)}{(\mathcal{R}^2 + T^2)^{3/2}}, \\ W_y &= n^2 y - \frac{(1-\mu)q_1 y}{r_1^3} - \frac{\mu q_2 y}{r_2^3} - \frac{M_b(y - 2k/n)}{(\mathcal{R}^2 + T^2)^{3/2}}. \end{aligned} \tag{7}$$

Here q_1 and q_2 are the radiation pressure factor for m_1 and m_2 respectively. We consider the same coordinate system in Luk'yanov (2009) where the origin of the rotational coordinate is the position of m_1, hence

$$\begin{aligned} r_1^2 &= x^2 + y^2, \\ r_2^2 &= (x-1)^2 + y^2. \end{aligned} \tag{8}$$

Here r_1 and r_2 are the distance of the third body from m_1 and m_2. Since the center of the disk–like structure is the point A, *i.e.* the point around which the primaries barycenter orbits, the radial distance of the infinitesimal mass becomes

$$\mathcal{R}^2 = (x-\mu)^2 + (y-2k/n)^2. \tag{9}$$

Equation 1 is similar to the equation of motion in Luk'yanov (2009) if the effects from radiation and disk–like structure are neglected.

3. EQUILIBRIUM POINTS

The equilibrium points are the position where the third body is motionless with respect to the primaries. By considering $\ddot{x} = \ddot{y} = \dot{x} = \dot{y} = 0$ into equation 5, we have

$$\begin{aligned} (x-\mu)\left[n^2 - \frac{M_b}{(\mathcal{R}^2+T^2)^{3/2}}\right] - \frac{(1-\mu)q_1 x}{r_1^3} - \frac{\mu q_2(x-1)}{r_2^3} &= 0, \\ y\left[n^2 - \frac{M_b}{(\mathcal{R}^2+T^2)^{3/2}}\right] - \frac{(1-\mu)q_1 y}{r_1^3} - \frac{\mu q_2 y}{r_2^3} + \frac{2kM_b}{n(\mathcal{R}^2+T^2)^{3/2}} &= 0. \end{aligned} \tag{10}$$

The position of equilibrium points are calculated by solving equation 5 for x and y.

3.1. QUASI-COLLINEAR POINTS

Conventionally the collinear points L_1, L_2, and L_3 are the solution located in the x-axis ($y = 0$) with the interval $1 < x < \infty$, $0 < x < 1$, and $-\infty < x < 0$, respectively. By considering $y = 0$ in equation 10, it is clear that the collinear equilibrium points only exist if $M_b = 0$ or $k = 0$. Nevertheless, we searched possible equilibrium points near x-axis ($y \approx 0$) when $M_b \neq 0$ and $k \neq 0$ by calculating the equilibrium points numerically. Hereafter we called these equilibrium points as quasi-collinear equilibrium points. The numerical values were obtained by solving equation 10 using a numerical algorithm in Mathematica.

In order to analyse the influence of each additional effect to the equilibrium point position, we vary the value of M_b, k, q_1, and q_2. Here we consider $\mu_0 = 0.3$, $t_0 = 0$, and $T = 0.2$. The resulting time dependence graphs can be seen in Figures 2, 3, and 4. The equilibrium points become quasi-collinear. They shifted slightly towards the $+y$ axis, due to the existence of mass variation and the disk like structure, which has the point A (that is not on the barycenter, nor is it anywhere in the x axis) as its center. According to Figure 2, the position of L_1, L_2, L_3 have been affected by

M_b. Higher M_b makes L_1, L_2, and L_3 position further away from x-axis. Meanwhile, higher k means that the position of L_1, L_2, and L_3 are shifted higher with respect to the original position as time increases (see Figure 3). As shown in Figure 4, by considering several values of q_1 and q_2, we found that the change of q_1 and q_2 also affects the position of L_1, L_2, and L_3.

3.2. TRIANGULAR POINTS

In order to find the position of triangular equilibrium points, we have to solve equation 10 by considering $y \neq 0$. We assume that the position of triangular points in the modified CRTBP is the perturbed version of classical case ($r_1 = 1; r_2 = 1$), *i.e.*

$$\begin{aligned} r_1 &= 1 + \epsilon_1, \\ r_2 &= 1 + \epsilon_2, \end{aligned} \tag{11}$$

where $\epsilon_{1,2} \ll 1$. By substituting equation 11 to equation 8, neglecting higher order of $\epsilon_{1,2}$, and solving for x and y, we have position of triangular equilibrium points as follows

$$\begin{aligned} x &= \frac{1}{2} + \epsilon_1 - \epsilon_2, \\ y &= \sqrt{\frac{3}{4} + \epsilon_1 + \epsilon_2}. \end{aligned} \tag{12}$$

Following Singh and Taura (2013), we consider equation 11 and equation 12 in equation 10. Hence, with additional expansion of k to the first order, we obtain

$$\begin{aligned} \epsilon_1 &= -\frac{1-q_1}{3} + \frac{M_b(1-2r_c)}{3(r_c^2+T^2)^{3/2}}, \\ \epsilon_2 &= -\frac{1-q_2}{3} + \frac{M_b(1-2r_c)}{3(r_c^2+T^2)^{3/2}}. \end{aligned} \tag{13}$$

Substituting equation 13 to Equation 12 yields the triangular points L_4 and L_5

$$x = \frac{1}{2} - \frac{q_2 - q_1}{3} \tag{14}$$

and

$$y = \pm\frac{\sqrt{3}}{2}\left(1 - \frac{2}{9}(2 - q_1 - q_2) + \frac{4}{9}\frac{M_b(1-2r_c)}{(r_c^2+T^2)^{3/2}}\right). \tag{15}$$

It can be seen that the triangular points for this system are identical (to the first order) with the constant primary mass counterpart, albeit with a (slow) time dependence.

Figure 2 shows the effect of M_b in the position of triangular points. We observe that the position of L_4 and L_5 is not symmetric due to the combination of disk–like structure and mass transfer. This asymmetric is larger when M_b is higher. We noted

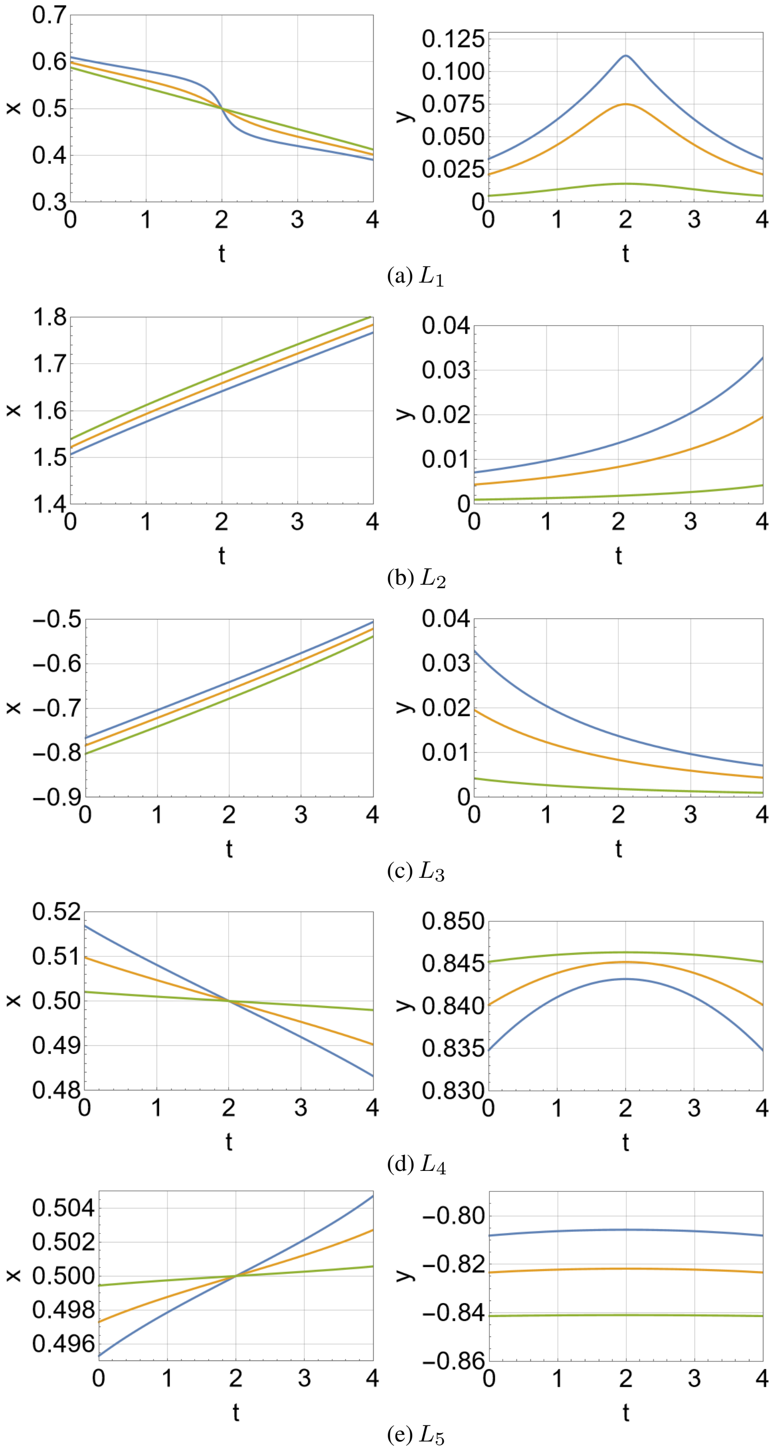
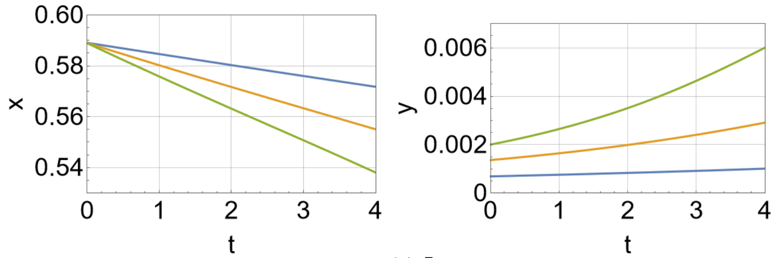
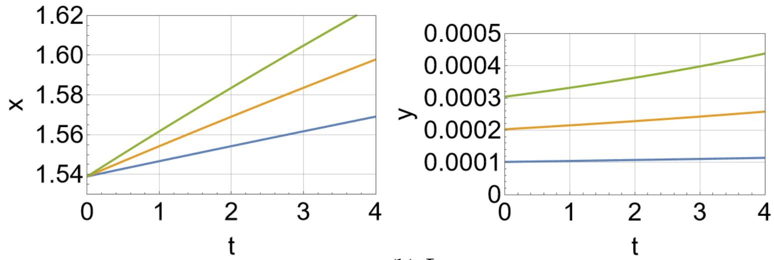


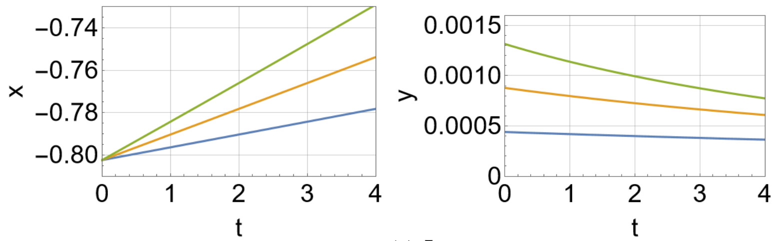
Fig. 2 – The position of equilibrium points for $M_b = 0.09$ (■), $M_b = 0.05$ (■), $M_b = 0.01$ (■). Here $k = 0.1$ and $q_1 = q_2 = 0.95$.



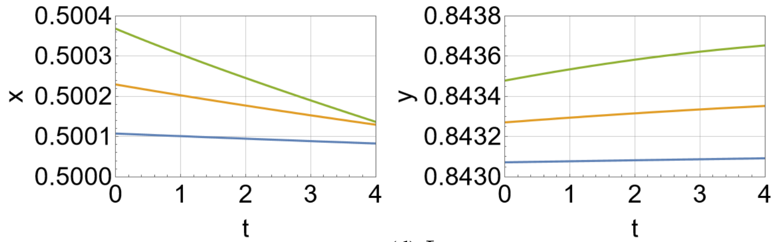
(a) L_1



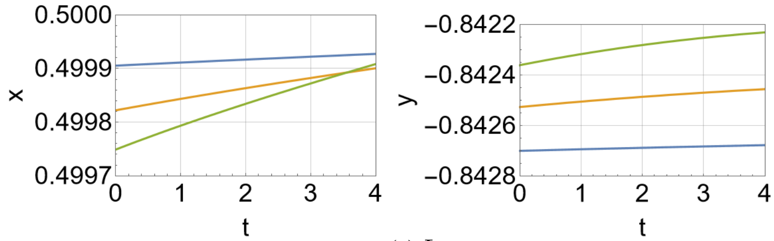
(b) L_2



(c) L_3



(d) L_4



(e) L_5

Fig. 3 – The position of equilibrium points for $k = 0.01$ (■), $k = 0.02$ (■), $k = 0.03$ (■). Here $M_b = 0.01$ and $q_1 = q_2 = 0.95$.

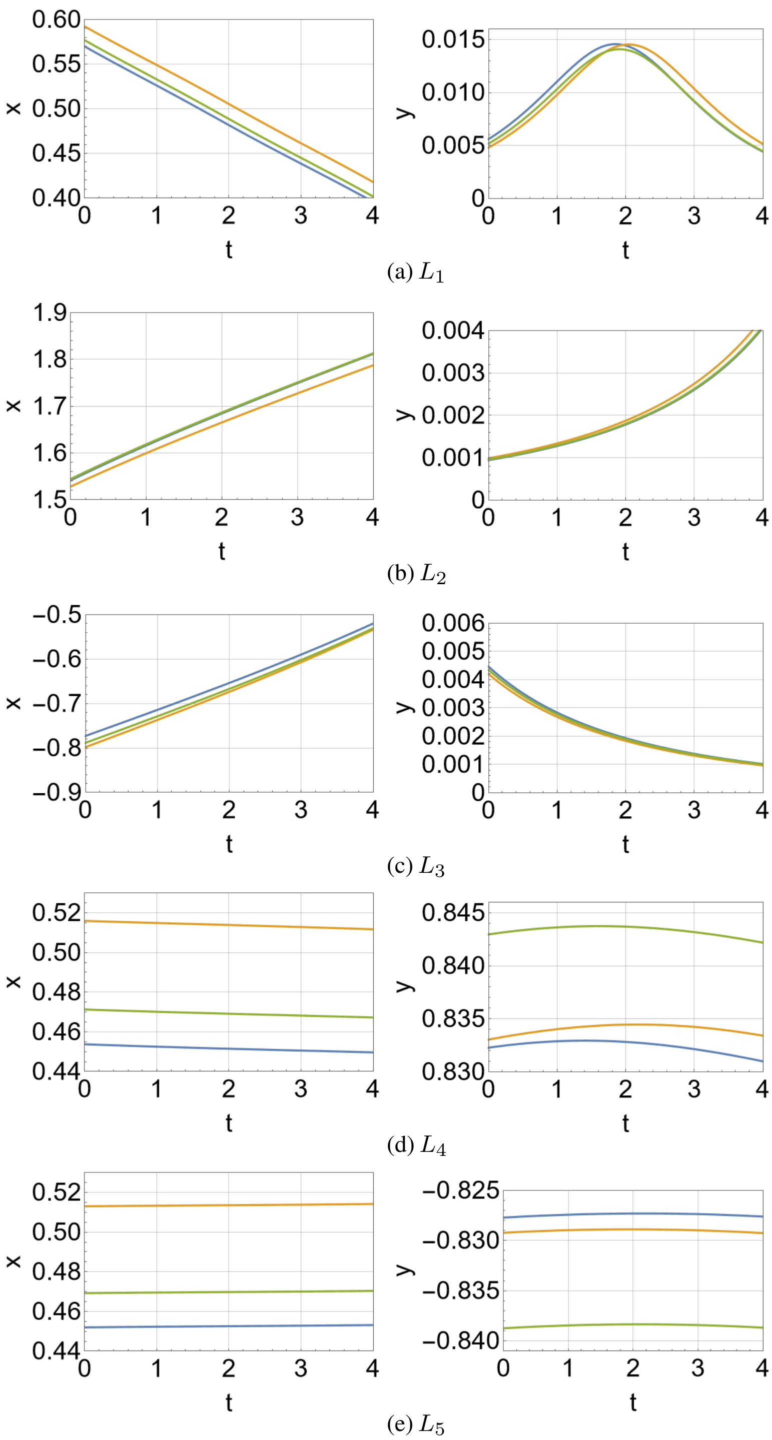


Fig. 4 – The position of equilibrium points for $q_1 = 0.85$; $q_2 = 0.99$ (■), $q_1 = 0.94$; $q_2 = 0.9$ (■), $q_1 = 0.9$; $q_2 = 0.99$ (■). Here $M_b = 0.01$ and $k = 0.1$.

also that the position of L_4 and L_5 is closer to the primaries with increasing M_b. In Figure 3, it is clear that the decreasing value of k makes L_4 closer to the primaries, in contrast with L_5. From Figure 4 we observe that radiation pressure has an impact on the location of triangular equilibrium points. The location of L_4 and L_5 are closer to the source of radiation pressure when the radiation pressure gets stronger, either for m_1 or m_2.

4. LINEAR STABILITY

The stability of equilibrium points are studied by introducing the perturbation in the equilibrium point (x_0, y_0), hence we define

$$\begin{aligned} x &= x_0 + \alpha, \\ y &= y_0 + \beta, \end{aligned} \tag{16}$$

where α and β are small displacements with respect to the equilibrium points. By substituting equation (16) into equation (5) and expanding the equation, we get

$$\begin{aligned} \ddot{\alpha} - 2n\dot{\beta} &= W^0_{xx}\alpha + W^0_{xy}\beta, \\ \ddot{\beta} + 2n\dot{\alpha} &= W^0_{yx}\alpha + W^0_{yy}\beta, \end{aligned} \tag{17}$$

where

$$\begin{aligned} W_{xx} &= n^2 + \frac{(1-\mu)q_1}{r_1^3}\left(-1 + \frac{3x^2}{r_1^2}\right) + \frac{\mu q_2}{r_2^3}\left(-1 + \frac{3(x-1)^2}{r_2^2}\right) + \frac{M_b}{(R^2+T^2)^{3/2}}\left(-1 + \frac{3(x-\mu)^2}{(R^2+T^2)}\right), \\ W_{yy} &= n^2 + \frac{(1-\mu)q_1}{r_1^3}\left(-1 + \frac{3y^2}{r_1^2}\right) + \frac{\mu q_2}{r_2^3}\left(-1 + \frac{3y^2}{r_2^2}\right) + \frac{M_b}{(R^2+T^2)^{3/2}}\left(-1 + \frac{3(y-2k/n)^2}{(R^2+T^2)}\right), \\ W_{xy} &= W_{yx} = \frac{3(1-\mu)q_1 xy}{r_1^5} + \frac{3\mu q_2(x-1)y}{r_2^5} + \frac{3M_b(x-\mu)(y-2k/n)}{(R^2+T^2)^{5/2}}. \end{aligned} \tag{18}$$

The characteristic equation is given by

$$\lambda^4 + \left(4n^2 - W^0_{xx} - W^0_{yy}\right)\lambda^2 + W^0_{xx}W^0_{yy} - \left(W^0_{xy}\right)^2 = 0. \tag{19}$$

The solution of this equation is given as follows

$$\lambda_i = \pm\sqrt{(-b \pm \sqrt{b^2 - 4c})/2}; \quad i = 1,2,3,4. \tag{20}$$

where $b = 4n^2 - W^0_{xx} - W^0_{yy}$ and $c = W^0_{xx}W^0_{yy} - (W^0_{xy})^2$. The stability of equilibrium points can be achieved when all λ_i are purely imaginary, otherwise we have an unstable equilibrium point.

Table **??** shows the characteristic roots (λ_i) of the collinear equilibrium points by considering several configurations of perturbing parameters. All λ_1 have the form real which signifies instability. In the range of mass parameter $0 < \mu < 1$ we found

Table 1 Characteristic roots of collinear equilibrium points with $\mu = 0.02$. We used $T = 0.2$ and $t_0 = 0$. Here i means $\sqrt{-1}$. λ_2 and λ_4 have the inverse sign of λ_1 and λ_3 respectively

Case	$1-q_1$	$1-q_2$	M_b	k	t	L_1		L_2		L_3	
						λ_1	λ_3	λ_1	λ_3	λ_1	λ_3
1	1	1	0	0.1	0	3.0140	$2.3861i$	2.0987	$1.8277i$	0.2277	$1.0169i$
	1	1	0	0.1	0.2	3.1535	$2.4749i$	1.9959	$1.7685i$	0.3204	$1.0329i$
	1	1	0	0.1	0.3	3.2054	$2.5081i$	1.9568	$1.7462i$	0.3574	$1.0407i$
2	0.05	0.03	0	0.1	0	2.8645	$2.2919i$	2.1797	$1.8750i$	0.2297	$1.0172i$
	0.05	0.03	0	0.1	0.2	3.0242	$2.3926i$	2.0553	$1.8026i$	0.3232	$1.0335i$
	0.05	0.03	0	0.1	0.3	3.0815	$2.4290i$	2.0102	$1.7767i$	0.3605	$1.0413i$
3	0.05	0.03	0.001	0.1	0	2.8640	$2.2921i$	2.1834	$1.8777i$	0.2292	$1.0181i$
	0.05	0.03	0.001	0.1	0.2	3.0242	$2.3931i$	2.0585	$1.8050i$	0.3230	$1.0344i$
	0.05	0.03	0.001	0.1	0.3	3.0817	$2.430i$	2.0133	$1.7791i$	0.3604	$1.0423i$
4	0.05	0.03	0.001	0.2	0	2.8634	$2.2916i$	2.1835	$1.8778i$	0.2279	$1.0178i$
	0.05	0.03	0.001	0.2	0.2	3.1302	$2.4604i$	1.9741	$1.7567i$	0.3935	$1.0497i$
	0.05	0.03	0.001	0.2	0.3	3.2118	$2.5125i$	1.9071	$1.7187i$	0.4533	$1.0646i$

that $b^2 - 4c > 0$ for L_1, L_2, and L_3. Consequently we have at least one positive real for the solution of the characteristic equation. Hence, the collinear equilibrium points are always unstable.

In the case of triangular equilibrium points, the stability is achieved when $0 < \mu < \mu_c$, where μ_c means the critical mass. Following Singh and Taura (2013), the critical mass is given as follows

$$\mu_c = \frac{1}{2}\left(1 - \sqrt{\frac{23}{27}}\right) - 2\frac{2 - q_1 - q_2}{27\sqrt{69}} + \left(\frac{3}{2} + \frac{(76 - 8r_c)(r_c^2 + T^2)}{27\sqrt{69}} - \frac{83 + 12r_c^2}{6\sqrt{69}}\right)\frac{M_b}{(r_c^2 + T^2)^{5/2}}. \tag{21}$$

It has to be noted that equation 21 differs from Singh and Taura (2013) since, in our case, we consider the mass transfer. Table 2 shows examples of characteristic roots in the stability of triangular equilibrium points. All cases have stable equilibrium points during $t = 0$ and unstable in $t = 0.2$ and $t = 0.3$. We noted that λ_1 and λ_3 in L_4 are similar to L_5 for the case 1 and case 2. However, due to the combination of disk–like structure and mass transfer effects, this similarity is not sound for case 3 and case 4. Since, in our case, μ depends on time, besides μ_c there exists also a so called critical

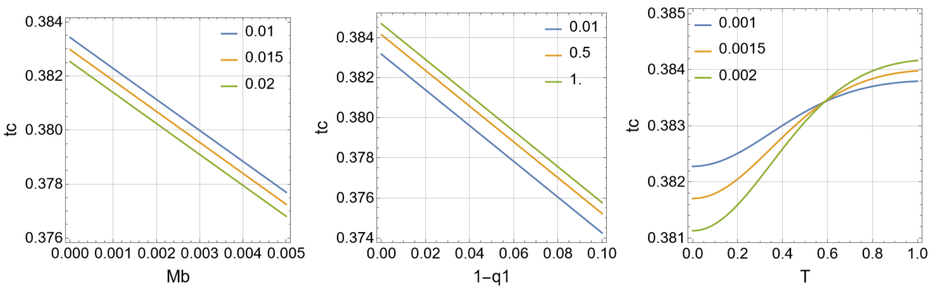


Fig. 5 – t_c as a function of (a) M_b, (b) q_1, and (c) T, for various values of (a) $1-q_1$, (b) T, and (c) M_b.

time (t_c) as follows

$$t_c = t_0 + (\mu_c - \mu_0)/k, \tag{22}$$

where $t < t_c$ means stable. Figure 5 shows the effect of perturbing parameters M_b, q_1, and T on t_c for the case of $k = 0.1$, $\mu_0 = 0.3$, and $t_0 = 3$. We noted that t_c becomes shorter when M_b and $1-q_1$ increase. In contrast, t_c is longer if T increases.

Table 2 Characteristic roots (λ_1 and λ_3) of triangular equilibrium points with $\mu = 0.02$. We used $T = 0.2$ and $t_0 = 0$. Here i means $\sqrt{-1}$. λ_2 and λ_4 have the inverse sign of λ_1 and λ_3 respectively

Case	$1-q_1$	$1-q_2$	M_b	k	t	L_4		L_5	
						λ_1	λ_3	λ_1	λ_3
1	1	1	0	0.1	0	$0.3961i$	$0.9182i$	$0.3961i$	$0.9182i$
	1	1	0	0.1	0.2	$0.0675+0.7103i$	$0.0675-0.7103i$	$0.0675+0.7103i$	$0.0675-0.7103i$
	1	1	0	0.1	0.3	$0.1820+0.7302i$	$0.1820-0.7302i$	$0.1820+0.7302i$	$0.1820-0.7302i$
2	0.05	0.03	0	0.1	0	$0.4005i$	$0.9163i$	$0.4005i$	$0.9163i$
	0.05	0.03	0	0.1	0.2	$0.0828+0.7119i$	$0.0828-0.7119i$	$0.0828+0.7119i$	$0.0828-0.7119i$
	0.05	0.03	0	0.1	0.3	$0.1889+0.7319i$	$0.1889-0.7319i$	$0.1889+0.7319i$	$0.1889-0.7319i$
3	0.05	0.03	0.001	0.1	0	$0.4018i$	$0.9167i$	$0.4006i$	$0.9174i$
	0.05	0.03	0.001	0.1	0.2	$0.0835+0.7127i$	$0.0835-0.7127i$	$0.0822+0.7126i$	$0.0822-0.7126i$
	0.05	0.03	0.001	0.1	0.3	$0.1892+0.7326i$	$0.1892-0.7326i$	$0.1888+0.7326i$	$0.1888-0.7326i$
4	0.05	0.03	0.001	0.2	0	$0.4074i$	$0.9139i$	$0.4007i$	$0.9174i$
	0.05	0.03	0.001	0.2	0.2	$0.2499+0.7504i$	$0.2499-0.7504i$	$0.2477+0.750i$	$0.2477-0.750i$
	0.05	0.03	0.001	0.2	0.3	$0.3263+0.7792i$	$0.3263-0.7792i$	$0.3252+0.7790i$	$0.3252-0.7790i$

5. CONCLUSIONS

We have studied the influence of mass transfer, disk–like structure, and radiation pressure, on the position and stability of CRTBP equilibrium points. In this system, we found there are five equilibrium points, where two of them are triangular equilibrium points, and the others are quasi-collinear equilibrium points. Unlike the classical collinear equilibrium points, we noted that L_1, L_2, and L_3 are slightly departed from x-axis since there exist the effects from disk–like structure and mass transfer. Moreover, the symmetry of L_4 and L_5 is broken when we consider the mass transfer and disk–like structure together. Furthermore, we found that the quasi-collinear equilibrium points remain unstable. The stability of triangular points depends on the initial mass parameter μ_0 as well as the time. Besides μ_c, we found there exists critical time, t_c, for achieving the stability of triangular points. The stability is achieved when $\mu < \mu_c$ and $t < t_c$.

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