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STRING CLOUD WITH QUARK MATTER AND VISCOUS FLUID ADMITTING CONFORMAL MOTION

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Abstract. This paper addresses the resolution of Einstein's field equations within a spherically symmetric, higher-dimensional space-time. The solutions incorporate strange quark matter connected to a string cloud with bulk viscous fluid, utilizing the assumption of a one-parameter group of conformal motions. Additionally, the characteristics of these solutions are explored within the context of higher-dimensional space-time.

Key words: bulk viscous fluid, quark matter, conformal motion.

1. INTRODUCTION

For the evaluation of the universe, there is a general assumption that, during the phase transition (as the universe passes through its critical temperature), the symmetry of the universe is spontaneously broken. Cosmic strings, being topologically stable objects, may have formed during a phase transition in the early universe. The investigation of cosmic strings and their associated physical processes has gained considerable attention (Vilenkin, 1981) due to their potential to induce density perturbations, leading to the formation of galaxies (Zel'dovich, 1980).

The initial success of string theory was in providing a consistent framework for quantum gravity, resembling General Relativity at macroscopic distances. As the stress-energy of a string can couple with the gravitational field, exploring the gravitational effects arising from strings becomes intriguing. Yilmaz (2005, 2006) delved into 5-D Kaluza-Klein cosmological models with quark matter attached to the string cloud and domain wall.

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Symmetries of relevant geometric or physical quantities are referred to as congruences, with conformal Killing vectors being particularly useful. This study imposes the condition that the space-time manifold admits a conformal Killing vector, providing deeper insights into the space-time geometry relevant to the astrophysical and cosmological realms (Harko *et al.*, 2004; Mars & Senovilla, 1994; Neves, 2006; Rahman *et al.*, 2010).

The investigation of inherited symmetries with a conformal Killing vector and spherical conformal Killing vector in spacetimes (perfect, anisotropic, viscous, and heat-conducting) has recently attracted interest. Herrera & Ponce de Leon (1985) studied conformal Killing vectors with a focus on perfect and anisotropic fluids. Yavuz & Yilmaz (1997) and Yilmaz *et al.* (1999) considered inheriting conformal and special conformal Killing vectors, as well as curvature inheritance symmetry in string cosmology (string cloud and string fluids), respectively. Baysal *et al.* (2002) explored conformal collineation in string cosmology.

This study considers the attachment of strange quark matter to the string cloud with bulk viscous fluid. Bodmer (1971) and Witten (1984) proposed two ways of forming strange matter: the quark-hadron phase transition in the early universe and the conversion of neutron stars into strange ones at ultrahigh densities.

As a gravitating object undergoes indefinite collapse, the end product is a singularity marked by the divergence of physical parameters like energy density. Considering the state of matter at ultra-high density beyond nuclear matter becomes relevant. Strange quark matter, consisting of u, d, and s quarks, is a possible state in such scenarios.

In the simplified version of the bag model, quarks are assumed to be massless and non-interacting. The quark pressure $P_q = \frac{\rho_q}{3}$ (ρ_q and the total energy density as, $\rho = \rho_q + B_c$. where B_c is the difference between the energy density of perturbative and non-perturbative quantum chromodynamics (QCD) vacuum (the bag constant). The total pressure is given by $p = p_q - B_c$. Basically, it is the bag constant B_c defining a consistent energy per unit volume linked to the space occupied by quarks.

Regarding pressure, B_c can be seen as the force pushing inward that is necessary to keep the quarks confined within the bag. It further ensures that $p = 0$ in vacuum. In the MIT bag model, such bag constant was incorporated as a phenomenological parameter to represent the energy expense of quark matter as compared to the confined vacuum state. The quark pressure decreases further with the increase in the value of B_c. The pressure within the gravitating object diminishes from its core to its surface, forming a decreasing function in relation to radial distance. At every radial point, this pressure counteracts its gravitational collapse.

The study assumes that space-time admits a one-parameter group of conformal motions, *i.e.*,

$$L_\xi g_{ab} = 2\psi g_{ab}\,,\ \psi = \psi(x^a), \tag{1}$$

where $L_\xi$signifies the Lie derivative along ξ^α and $\psi(x^a)$ is the conformal factor. In particular, ζ is a special conformal Killing vector if $\psi_{;ab} = 0$ and $\psi_{,a} = 0$. Other subcases include homothetic vectors (HV) if ξ is constant. Here (;) and (,) represent the covariant and ordinary derivatives, respetively. The vector ξ generates the coformal symmetry whereas the metric g is conformally mapped onto itself along ξ. Neither ξ nor ξ are necessarily static, as evident in the case of static metric (Bohmer *et al.* 2007, 2008).

This study examines charged strange quark matter attached to the string cloud with bulk viscous fluid in a five-dimensional spherically symmetric space-time admitting a one-parameter group of conformal motion. It extends the work obtained earlier by Yavuz *et al.* (2005) for four-dimensional space-time.

The organization of our work is envisaged as follows: In Section 2, we derive the five-dimensional Einstein field equations for charged strange quark matter attached to the string cloud in spherically symmetric space-time. In Section 3, we solve the Einstein field equations for the same matter using the idea of conformal motion depending on a conformal vector, *i.e.*, $\psi(x^a)$. Finally, in Section 4, we summarize our results.

2. THE METRIC AND FIELD EQUATIONS

We examine the spherically symmetric line element in five dimensions, expressed as

$$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2 d\Omega^2, \tag{2}$$

with

$$d\Omega^2 = d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\theta_3^2$$
$$x^{1,2,3,4} = r, \theta_1, \theta_2, t.$$

The total energy-momentum tensor T_{ab} is postulated as the sum of two components, T^{Q}_{ab} and T^{E}_{ab}, for the contributions from the string cloud and electromagnetic fields, respectively, *i.e.*,

$$T_{ab} = T^{S}_{ab} + T^{E}_{ab}. \tag{3}$$

The energy-momentum tensor for the string cloud, as proposed by Letelier (1979) and Stachel (1980), is expressed as:

$$T_{ab} = \rho U_a U_b - \rho_s X_a X_b, \tag{4}$$

In this context, where ρ represents the rest energy for the string cloud with attached particles and ρ_s denotes the string tension density, their relationship is defined by:

$$\rho = \rho_p + \rho_s, \tag{5}$$

where ρ_p denotes the particle energy density.

Recognizing the freedom of strings to vibrate, various vibration modes of the string correspond to different types of particles, as these modes are perceived as distinct masses or spins. Consequently, we will substitute quarks for particles within the string cloud, specifically focusing on the energy density of strange quark matter instead of particle energy density. Under this consideration, equation (5) yields:

$$\rho = \rho_q + \rho_s + B_c. \tag{6}$$

By substituting equation (6) into equation (4), we obtain the expression for a quark attached to the string within the string cloud with bulk viscous fluid:

$$T^{S}_{ab} = (\rho_q + \rho_s + B_c) U_a U_b - \rho_s X_a X_b, \tag{7}$$

In this context, where U^a denotes the five velocity ($U^a = \delta^a_1 e^{\frac{\beta}{2}}$), and X^a represents the unit spacelike vector in the radial direction ($X^a = \delta^a_1 e^{\frac{\lambda}{2}}$) signifying the direction of strings within the cloud, *i.e.*, the direction of anisotropy, the expression becomes:

$$T^{E}_{ab} = -\frac{1}{4\pi}(F^{c}_{a} F_{bc} - \frac{1}{4} g_{ab} F_{\alpha\beta} F^{\alpha\beta}). \tag{8}$$

In this equation, F_{ab} represents the electromagnetic field tensor, defined in terms of the five potential A_0 as:

$$F_{ab} = A_{b;a} - A_{a;b}.$$

We adopt the gauge for the electromagnetic field given by

$$A_a(0,0,0,\phi(r)).$$

The Einstein-Maxwell equations can be formulated as:

$$R_{ab} - \frac{1}{2} R g_{ab} = 8\pi T_{ab}, \tag{9}$$

$$F_{ab;c} + F_{bc;a} + F_{ca;b} = 0, \tag{10}$$

$$F_{ab;b} = -4\pi J^a, \tag{11}$$

where J^a is the five current density that becomes $J^a = \bar{\rho}_e U^a$, $\bar{\rho}_e$ being the proper charge density of the fluid.

Utilizing the line element (2) and the field equations (9)–(11), the expression for the Einstein field equations is:

$$8\pi\rho + E^2 = -e^{-\lambda}\left[\frac{3}{r^2} - \frac{3\lambda'}{2r}\right] + \frac{3}{r^2}, \tag{12}$$

$$8\pi\rho - E^2 = e^{-\lambda}\left[\frac{3\nu'}{2r} + \frac{3}{r^2}\right] - \frac{3}{r^2}, \tag{13}$$

$$E^2 = \frac{e^{-\lambda}}{2}\left[\nu'' - \frac{\nu'\lambda'}{2} + \frac{\nu'^2}{2} - \frac{(\lambda' - \nu')}{r} + \frac{1}{r^2}\right] - \frac{1}{r^2}, \tag{14}$$

$$\left[r^3 E(r)\right]' = 4\pi\rho_e r^3, \tag{15}$$

In the provided expression, the prime $(')$ indicates differentiation with respect to r, and E represents the conventional electric field intensity, defined as:

$$F_{01}F^{01} = -E^2, \;\; E(r) = -e^{-(\nu+\lambda)/2}\phi'(r),$$

$$\phi'(r) = F_{10} = -F_{01}. \tag{16}$$

The charge density ρ_e defined in equation (15) is connected to the proper charge density of the fluid $\bar{\rho}_e$ by

$$\rho_e = \bar{\rho}_e e^{\lambda/2}. \tag{17}$$

3. FIELD EQUATIONS SOLVED

To obtain a deterministic solution, we assume that the space-time allows for a one-parameter group of conformal motions, *i.e.*,

$$\mathcal{L}_\xi g_{ab} = \xi_{a;b} + \xi_{b;a} = 2\psi g_{ab}, \tag{18}$$

Here, ψ is an arbitrary function of r. From equations (1) and (18) considering spherical symmetry, we derive the following expressions:

$$\xi^1\nu' = \psi, \quad \xi^1 = \frac{\psi r}{2}, \tag{19}$$

$$\lambda'\xi' + 2\xi^1_{,1} = \psi, \quad \xi^4 = C_1 = \text{constant}, \tag{20}$$

Here, a comma $(')$ indicates a partial derivative with respect to r. From equations (19)–(20), we obtain

$$e^\nu = C_2^2 r^2, \;\; e^\lambda = \left(\frac{C_3^2}{\psi}\right)^2, \tag{21}$$

$$\xi^a = \frac{\psi r}{2}\delta_1^a + C_1\delta_4^a, \tag{22}$$

where C_1 and C_2 represent constants of integration.

By substituting equation (21) into the field equations (12)–(14), we obtain:

$$\rho + E^2 = \frac{3}{r^2}[1 - \frac{\psi^2}{C_3^2}] - \frac{3\psi\psi^{'}}{C_3^2 r}, \tag{23}$$

$$\rho_s - E^2 = \frac{3}{r^2}\left[\frac{2\psi^2}{C_3^2} - 1\right], \tag{24}$$

$$E^2 = \frac{3\psi\psi^{'}}{C_3^2 r} + \frac{3\psi^2}{C_3^2 r^2} - \frac{1}{r^2}, \tag{25}$$

From equations (23)–(25), we get

$$\rho = -\frac{6\psi\psi'}{C_3^2 r} - \frac{6\psi^2}{C_3^2 r^2} + \frac{1}{r^2}, \tag{26}$$

$$\rho_s = -\frac{3\psi\psi'}{C_3^2 r} - \frac{9\psi^2}{C_3^2 r^2} + \frac{4}{r^2}, \tag{27}$$

$$E^2 = \frac{3\psi\psi'}{C_3^2 r} + \frac{3\psi^2}{C_3^2 r^2} - \frac{1}{r^2}. \tag{28}$$

From equation (6), we have

$$\rho_p = -\frac{3\psi\psi'}{C_3^2 r} + \frac{3\psi^2}{C_3^2 r^2}. \tag{29}$$

Utilizing equations (21), the line element (2) simplifies to:

$$ds^2 = C_2^2 r^2 dt^2 - \frac{C_3^2}{\psi^2} dr^2 - r^2 d\Omega^2. \tag{30}$$

As per equations (26)–(29), various solutions can be derived by specifying different choices for $\psi(r)$. Rather than arbitrarily assuming the function $\psi(r)$, we will explore specific solutions in the following.

3.1. CASE I : IF $\psi = c_4 r$

For this scenario, from equations (26)–(29), the results are:

$$\rho = \rho_s = \frac{4}{r^2} - \frac{12C_4^2}{2C_3^2}, \tag{31}$$

$$E^2 = -\frac{1}{r^2} + \frac{6{C_4}^2}{C_3^2}, \tag{32}$$

$$\rho_p = \rho - \rho_s = 0, \tag{33}$$

where C_4 is an integrating constant.

Let us consider that the charged sphere extends to a radius r_0. Then, for $r > r_0$, the solution of the Einstein-Maxwell equation is given by the Reissner-Nordstrom metric:

$$ds^2 = \left[1 - \frac{2M}{r} + \frac{q^2}{r^2}\right] dt^2 - \left[1 - \frac{2M}{r} + \frac{q^2}{r^2}\right]^{-1} dr^2 - r^2 d\Omega^2 \tag{34}$$

and the radial electric field is

$$E = \frac{q}{r^3}, \tag{35}$$

where M and q are the total mass and charge respectively of the fluid.

To match the line element (30) with the Reissner-Nordstrom metric across the boundary $r = r_0$, we require continuity of gravitational potential g_{ab} at $r = r_0$:

$$\left(C_2 r_0\right)^2 = \left(\frac{\psi}{C_3}\right)^2 = 1 - \frac{2M}{r_0} + \frac{q^2}{r_0^2}, \tag{36}$$

We also require the continuity of the electric field, leading to:

$$E(r_0) = \frac{q}{r_0^3}, \tag{37}$$

From equations (32) and (35), we get

$$\frac{q^2}{r_0^3} = 6\left(\frac{C_4}{C_3}\right)^2 - \frac{1}{r_0^2}, \tag{38}$$

Putting this expression back into equation (33), we obtain:

$$\frac{M}{r_0} = \frac{1}{2} + 3r_0^3\left(\frac{C_3}{C_4}\right)^2 - \frac{r_0}{2}, \tag{39}$$

The effective mass of the string cloud from equation (39), up to a 10 km radius (using the values of $C_3 = 0.1, C_4 = 1$), is found to be 172.8814 $M_\odot$, where $M_\odot$ is the solar mass.

Also, from equations (34) and (35), we get:

$$M = \frac{r_0}{2} + \frac{q^2}{2r_0}. \tag{40}$$

If $q = 0$ in equation (40), we get the total mass for a non-charged black string cloud, *i.e.*, Schwarzschild-like black string cloud.

From equations (36), (38) and (39), we get:

$$\psi = \pm\sqrt{3C_4^2 r_0\left(1 - \frac{r_0^2}{2}\right) + C_3^2\left(r_0 - \frac{1}{r_0}\right)}. \tag{41}$$

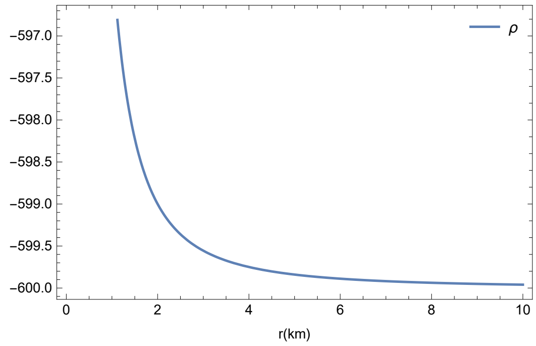

Fig. 1 – $\rho(r)$ is shown against r. We use constants $C_3 = 0.1$ and $C_4 = 1$.

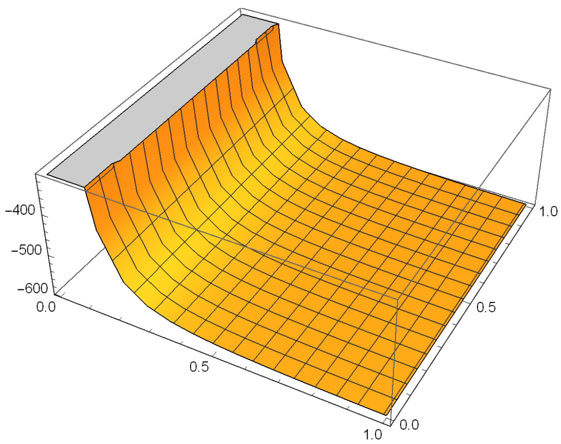

Fig. 2 – $\rho(r)$ is shown against r. We use constants $C_3 = 0.1$ and $C_4 = 1$.

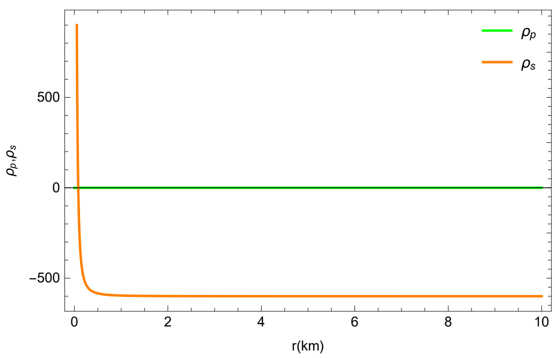

Fig. 3 – (ρ_p, ρ_s) are shown against r. We use constants $C_3 = 0.1$ and $C_4 = 1$.

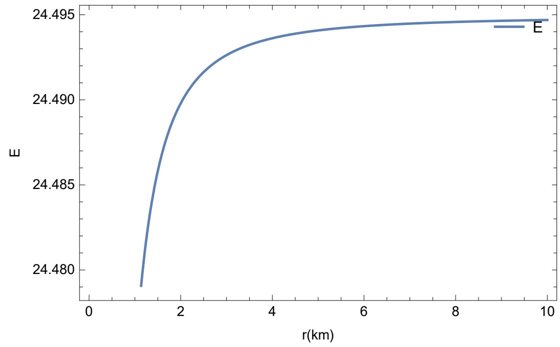

Fig. 4 – $E(r)$ is shown against r. We use constants $C_3 = 0.1$ and $C_4 = 1$.

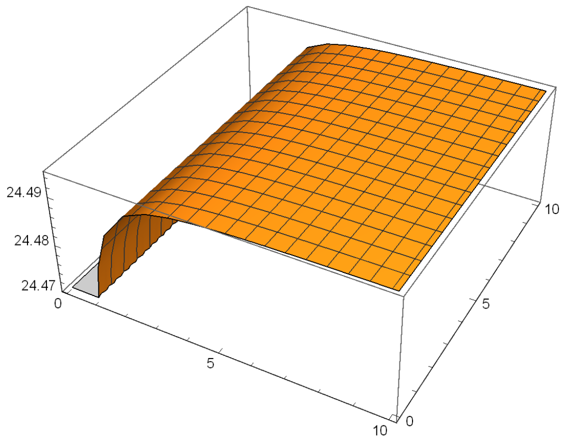

Fig. 5 – $E(r)$ is shown against r. We use constants $C_3 = 0.1$ and $C_4 = 1$.

3.2. CASE II : $\psi = \sqrt{\frac{4C_3^2}{9} + \frac{C_5}{r^6}}$

In this scenario, as per equations (26)–(29), we obtain:

$$\rho = \rho_p = \frac{4}{3r^2} + \frac{12C_5}{C_3^2 r^8}, \rho_s = 0 \tag{42}$$

$$\rho_q = \frac{4}{3r^2} + \frac{12C_5}{C_3^2 r^8} - B_c \tag{43}$$

$$E^2 = \frac{1}{3r^2} - \frac{5C_5}{C_3^2 r^8}, \tag{44}$$

where C_5 is integrating constant.

Figures 4, 5, and 6 suggest that strange quark matter reduces the energy of the string cloud.

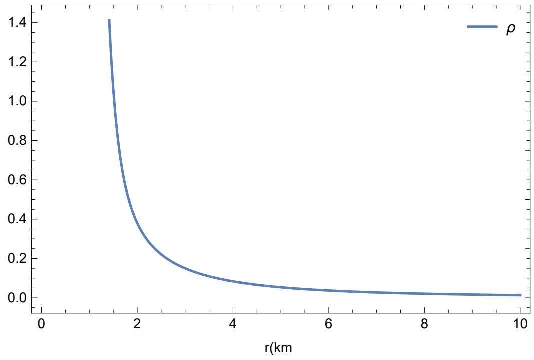

Fig. 6 – $\rho(r)$ is shown against r. We use constants $C_3 = 0.1$ and $C_5 = 0.01$.

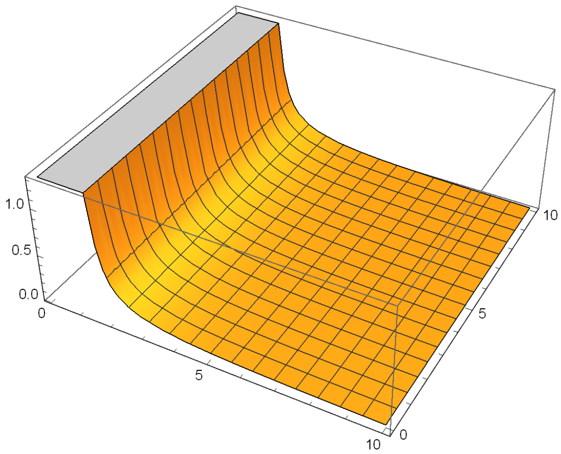

Fig. 7 – $\rho(r)$ is shown against r. We use constants $C_3 = 0.1$ and $C_5 = 0.01$.

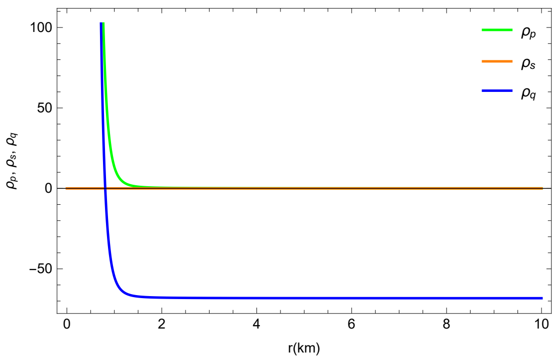

Fig. 8 – (ρ_p, ρ_s, ρ_q) are shown against r. We use constants $C_3 = 0.1$, $C_5 = 0.01$ and $B_c = 68.25$.

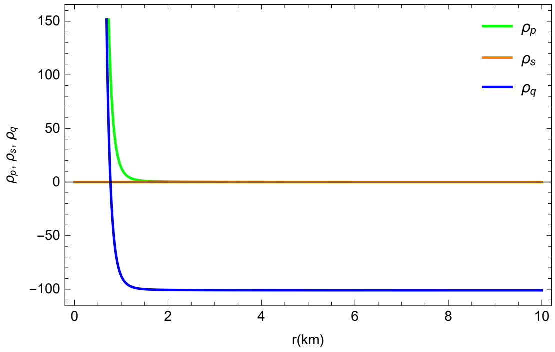

Fig. 9 – (ρ_p, ρ_s, ρ_q) are shown against r. We use constants $C_3 = 0.1$, $C_5 = 0.01$ and $B_c = 101.01$.

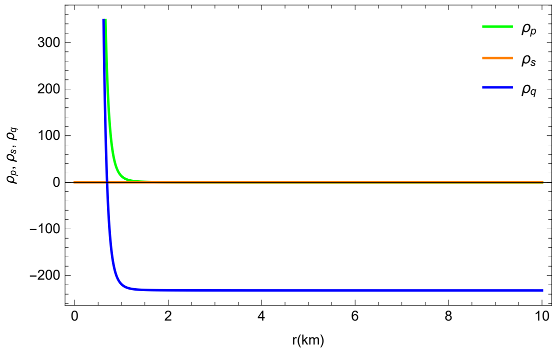

Fig. 10 – (ρ_p, ρ_s, ρ_q) are shown against r. We use constants $C_3 = 0.1$, $C_5 = 0.01$ and $B_c = 232.05$.

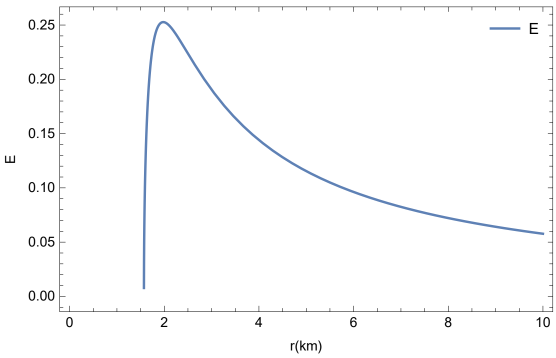

Fig. 11 – $E(r)$ is shown against r. We use constants $C_3 = 0.1$ and $C_5 = 0.01$.

3.3. CASE III : $\psi = \sqrt{\frac{1}{3}C_3^2 + \frac{C_6}{r^2}}$

In this case, as per equations (26)–(29), the results are:

$$\rho = \frac{2}{r^2}, \rho_s = \frac{1}{r^2} - \frac{6C_6}{C_3^2 r^4}, E^2 = 0 \tag{45}$$

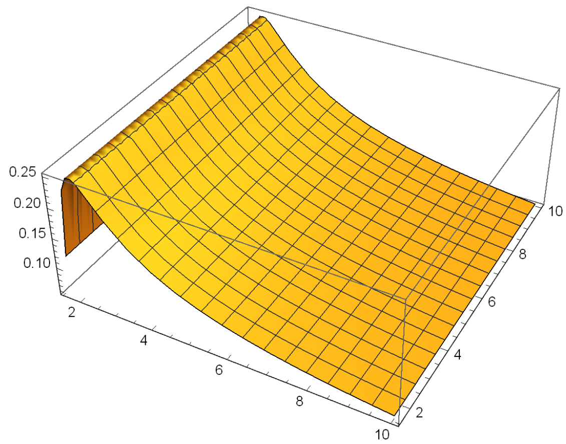

Fig. 12 – $E(r)$ is shown against r. We use constants $C_3 = 0.1$ and $C_5 = 0.01$.

$$\rho_p = \frac{1}{r^2} + \frac{6C_6}{C_3^2 r^4}, \tag{46}$$

$$\rho_q = \frac{1}{r^2} + \frac{6C_6}{C_3^2 r^4} - B_c, \tag{47}$$

Here, C_6 is an integrating constant.

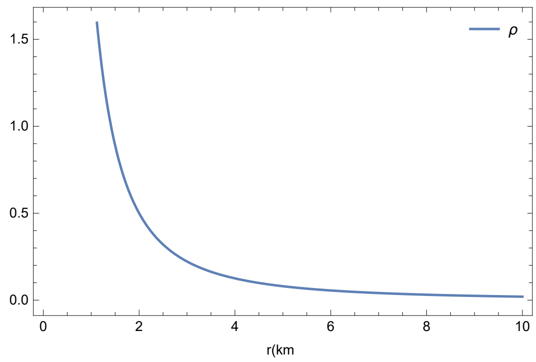

Fig. 13 – $\rho(r)$ is shown against r. We use constants $C_3 = 0.1$ and $C_6 = 0.1$.

Figures 10, 11 and 12 also suggest that strange quark matter reduces the energy of the string cloud.

Figure 20 indicates that the charge of the fluid is maximum at the surface of the string cloud.

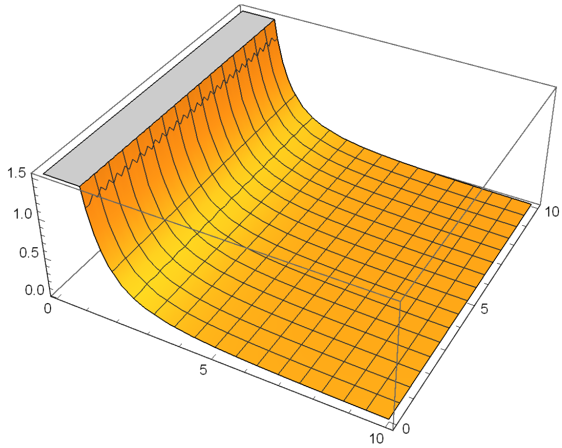

Fig. 14 – $\rho(r)$ is shown against r. We use constants $C_3 = 0.1$ and $C_6 = 0.1$.

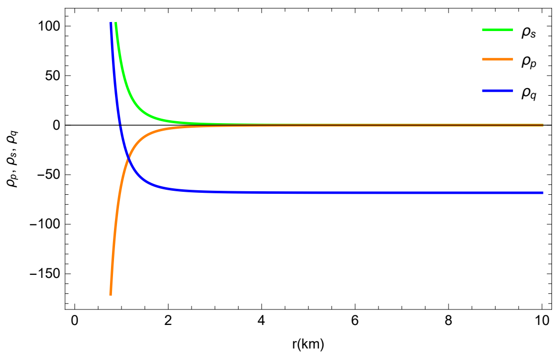

Fig. 15 – (ρ_p, ρ_s, ρ_q) are shown against r. We use constants $C_3 = 0.1$, $C_6 = 0.1$ and $B_c = 68.25$.

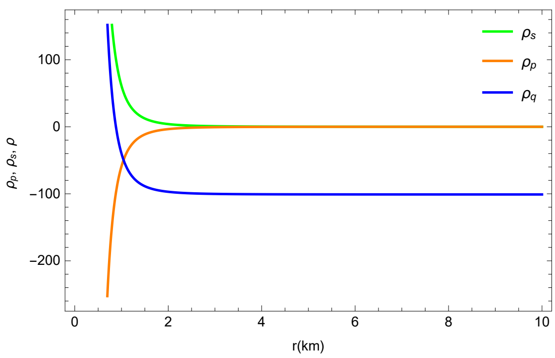

Fig. 16 – (ρ_p, ρ_s, ρ_q) are shown against r. We use constants $C_3 = 0.1$, $C_6 = 0.1$ and $B_c = 101.01$.

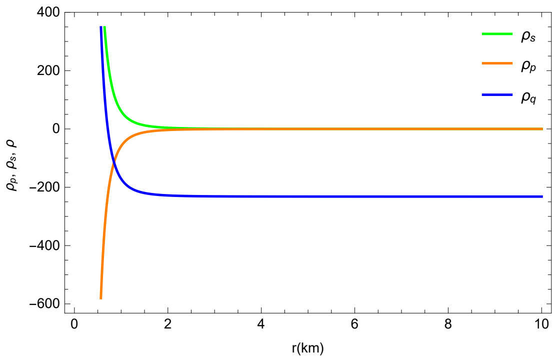

Fig. 17 – (ρ_p, ρ_s, ρ_q) are shown against r. We use constants $C_3 = 0.1$, $C_6 = 0.1$ and $B_c = 232.05$.

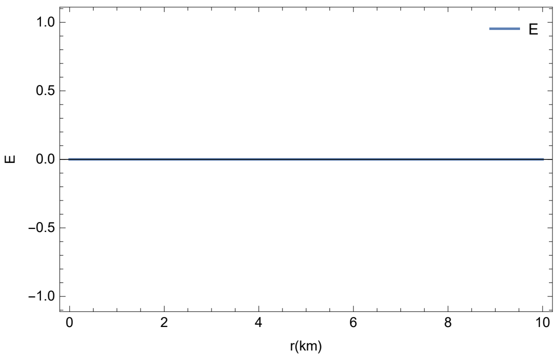

Fig. 18 – $E(r)$ is shown against r. We use constants $C_3 = 0.1$ and $C_6 = 0.1$.

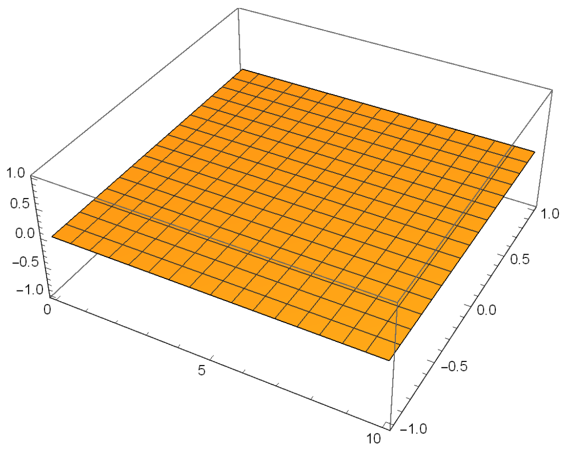

Fig. 19 – $E(r)$ is shown against r. We use constants $C_3 = 0.1$ and $C_6 = 0.1$.

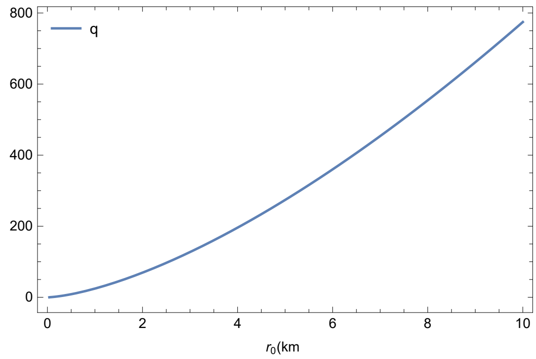

Fig. 20 – $q(r)$ is shown against r. We use constants $C_3 = 0.01$ and $C_4 = 1$.

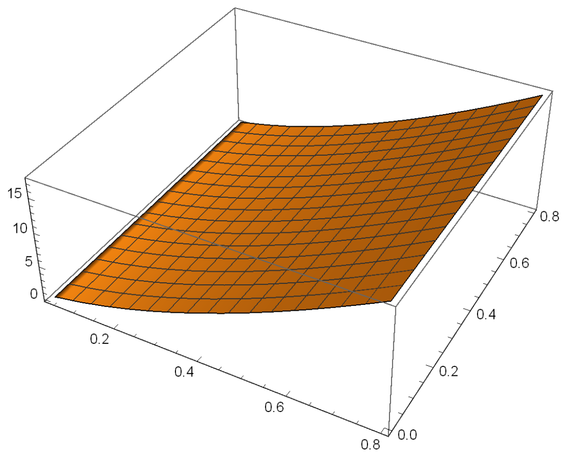

Fig. 21 – $q(r)$ is shown against r. We use constants $C_3 = 0.01$ and $C_4 = 1$.

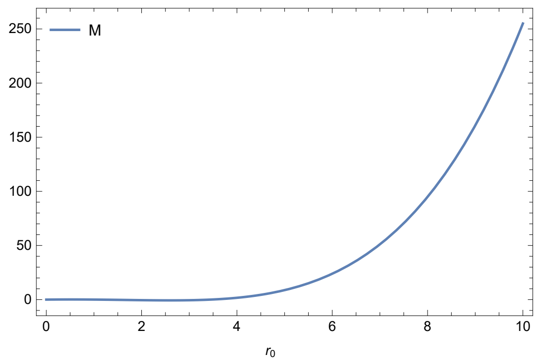

Fig. 22 – $M(r)$ are shown against r. We use constants $C_3 = 0.1$ and $C_4 = 1$.

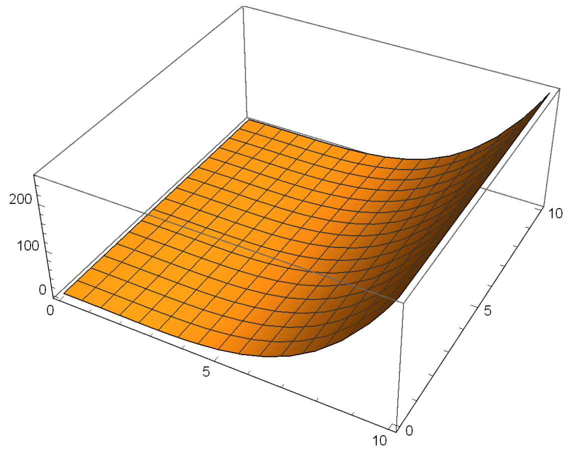

Fig. 23 – $M(r)$ are shown against r. We use constants $C_3 = 0.1$ and $C_4 = 1$.

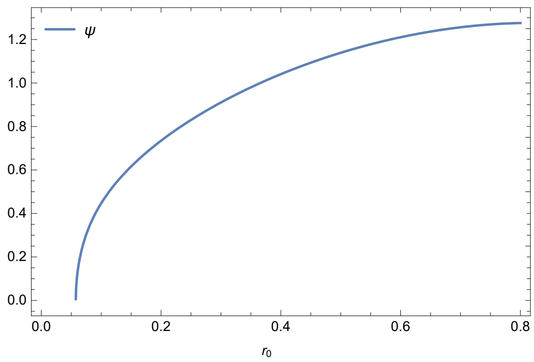

Fig. 24 – $\psi(r)$ is shown against r. We use constants $C_3 = 0.1$ and $C_4 = 1$.

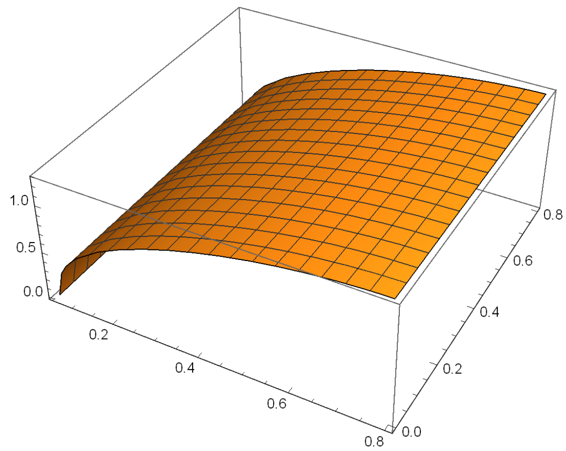

Fig. 25 – $\psi(r)$ is shown against r. We use constants $C_3 = 0.1$ and $C_4 = 1$.

4. CONCLUSION

In this paper, we have investigated charged strange quark matter attached to a string cloud with bulk viscous fluid within a spherically symmetric space-time that allows for a one-parameter group of conformal motions in the context of five-dimensional space-time. We observed that our findings in five dimensions exhibit similarities with the results obtained by Yavuz *et al.* (2005) in four dimensions. It can be deduced that the results of Yavuz *et al.* (2005) correspond to our results by assigning appropriate values to the relevant functions.

By studying our systems, scientists can gain insights into the dynamics and properties of the quark matter and viscous fluid interactions within the string cloud. Understanding how such string clouds behave in astrophysical environments, such as neutron star mergers or supernova explosions, can provide crucial information about the role of quark matter and viscous fluids in these extreme events. Investigating the role of string clouds with quark matter and viscous fluids in the early universe can shed light on the cosmological evolution and structure formation processes, helping to refine our understanding of the universe's history. Moreover in high-energy particle collision experiments, the presence of quark matter and viscous fluids within string clouds could manifest as unique particle signatures, providing experimental evidence to validate theoretical models.

Our study is helpful in observing compact objects such as neutron stars and black holes that are surrounded by string clouds. It could reveal distinctive observational features associated with the interaction between quark matter, viscous fluids, and the surrounding environment. Studying the conformal motion of the string cloud and its effects on nearby matter and radiation can provide insights into the nature of conformal symmetry breaking and its implications for fundamental physics. Overall, observational studies of string clouds incorporating quark matter and viscous fluids offer a rich avenue for exploring fundamental physics, astrophysics, and cosmology, providing valuable data to test and refine theoretical models of matter under extreme conditions.

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