

DARK MATTER AND RADIATION PRODUCTION DURING WARM INFLATION IN A CURVED UNIVERSE: AN IRREVERSIBLE THERMODYNAMIC APPROACH

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Abstract. We investigate the creation of dark matter particles as a result of the decay of the scalar field in the framework of warm inflationary models, by using the irreversible thermodynamics of open systems with matter creation/annihilation. We consider the scalar fields, radiation and dark matter as an interacting three component cosmological fluid in a homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker (FLRW) Universe, in the presence of the curvature terms. The thermodynamics of open systems as applied together with the gravitational field equations to the three component cosmological fluid leads to a generalization of the elementary scalar field-radiation interaction model, which is the theoretical basis of warm inflationary models. Moreover, the decay (creation) pressures describing matter production are explicitly considered as parts of the cosmological fluid energy-momentum tensor. A specific theoretical model, describing coherently oscillating scalar waves, is considered. In particular, we investigate the role of the curvature terms in the dynamical evolution of the early Universe, by considering numerical solutions of the gravitational field equations. Our results indicate that despite the fact that the Universe becomes flat at the end of the inflationary era, the curvature terms, if present, may still play an important role in the very first stages of the evolution of the Universe.

Key words: cosmology, warm inflation, irreversible thermodynamics, curvature effects.

1. INTRODUCTION

The question of the homogeneity settled over far apart regions in space, respectively the queries concerning the horizon and the flatness problems of the Universe, are beautifully answered by the theory of inflation, introduced in Guth (1981). Alan Guth's "old inflation" requires the existence of a scalar field, whose energy-momentum tensor mimics that of an ideal fluid. For a detailed discussion of the properties of scalar fields see Mukhanov (2005) and Nojiri *et al.* (2017), respectively. Later on, a *new inflation* scenario was proposed, in which the self-interacting

potential $V(\phi)$ of the scalar field was set to be nearly flat at its minimum, where it undergoes oscillatory fluctuations (Linde, 1982; Albrecht and Steinhardt, 1982). Due to multiple complications raised by this model, the chaotic inflationary scenario was developed in Linde (1983) and Linde (1994), respectively. In the chaotic inflation model one considers a region in space where at the initial time $t = t_0$ the scalar field is very large, and approximately homogeneous. The energy-momentum tensor of the scalar field is dominated by the large potential, thus leading to an equation of state $p_\phi \approx -\rho_\phi$, and to an inflationary expansion.

The above mentioned inflationary models are *cold inflation* models, in which the amplitude of the oscillating scalar field decreases, due to the expansion of the Universe, and because of an energy loss. As the Universe enters a cold, low-energy phase, a reheating phase, describing the production of Standard Model particles after the phase of accelerated expansion, is necessary, with the elementary particle production reaching thermal equilibrium at a certain reheating temperature. A slow reheating was effectively studied in chaotic inflation, (Albrecht *et al.*, 1982; Kofman *et al.*, 1994; Antusch *et al.*, 2015). Hence, the slow-roll inflation describes the early exponential expansion of the Universe and the reheating phase as two different processes.

The remnants of inflation can be observed in the temperature anisotropies of the Cosmic Microwave Background Radiation, by measuring the temperature fluctuations due to the variations of the matter density throughout the universe. The theoretical models in which the density perturbations have their origin in the quantum fluctuations of the scalar field, are known to run into some difficulties, as pointed out, for example, in Berera (1995). Henceforth, a novel theory of inflation emerged, starting with the works by Berera and Fang (1995), Berera (1995) and Berera *et al.* (2009), respectively, namely, the *warm inflation* scenario, in which the scalar field interacts dynamically with other fields through thermal dissipation, thus removing the quantum problem. Therefore, in the warm inflation proposal, the thermal fluctuations dominate over the quantum disturbances (Berera and Fang, 1995).

It was pointed out that warm inflation agrees neatly with COBE's measurements of temperature anisotropies in the CMB radiation (Berera, 1996), as opposed to the reheating model, which is not as consistent with observational data. In the warm inflationary scenario the transition from the scalar field-dominated Universe to the radiation era is accomplished by a dissipation coefficient Γ , inserted in the scalar field's equation of motion, which accounts for a continuous energy transfer through its decay. The scalar field and the radiation form together a cosmological fluid, for which the entropy perturbations have been studied in De Oliveira and Jorás (2001). Various aspects of the warm inflationary Universe have been considered throughout the literature in Gleisler and Ramos (1994), Hall *et al.* (2004), Liddle and Barrow (1994), Stewart (2002), and Bastero-Gil *et al.* (2011), respectively.

In the inflationary models the scalar field decays into matter, and thus particles are created. For a long time this process was investigated almost exclusively quantum mechanically. On the other hand, a thermodynamic approach to the adiabatic production of particles was introduced in Prigogine *et al.* (1988), and further developed in Calvão *et al.* (1992) and Zimdahl *et al.* (1996), respectively, where a constant entropy per particle was imposed during matter production. The cosmological fluid in the presence of particle creation was described thermodynamically in Prigogine *et al.* (1988), where the role played by the entropy in this process was pointed out. The decaying/creation process is assumed to be irreversible, allowing only for an energy flow from the scalar field to the newly created matter constituents. Henceforth, the particle creation in the framework of the irreversible thermodynamics of open systems has been applied for the study of various cosmological processes in Qiang *et al.* (2007), Modak and Singleton (2012), Harko and Lobo (2013), Chakraborty (2014), Chakraborty and Saha (2014), Lima and Baranov (2014), Harko (2014), Nunes and Pan (2016), Lima *et al.* (2016), Pigozzo *et al.* (2016), Su *et al.* (2017), Harko and Sheikahmadi (2020), Harko and Sheikahmadi (2021), and Pinto *et al.* (2022), respectively. Thus, in Harko and Lobo (2013), the interaction between dark matter and dark energy was modeled by using an approach based on the thermodynamics of irreversible processes, while in Su *et al.* (2017), the reheating theory was investigated by considering a three-component cosmological fluid.

The interaction between scalar fields and radiation in the framework of warm inflationary models was considered, for a homogeneous, spatially flat and isotropic FLRW Universe, in Harko and Sheikahmadi (2020). By assuming that the scalar fields and radiation form an interacting two component cosmological fluid, one can apply to this system the physical formalism of the irreversible thermodynamics of open systems in the presence of matter creation/annihilation. The theoretical predictions of the warm inflationary scenario with irreversible particle production were also compared with the Planck 2018 observational data, and thus constraints on the free parameters of the model were obtained.

It is the goal of the present paper to extend the warm inflationary scenario in three directions. First, since particle creation/decay processes represent dissipative irreversible processes, their proper formulation must be done in the framework of the thermodynamics of open system, as pointed out in Harko and Sheikahmadi (2020). Secondly, we will consider that not only radiation, but also a dark matter component, interacting with the scalar field and the radiation, is created during the initial stages of the expansion. Thirdly, we will assume that the Universe was not flat at its birth, but it may have had an (arbitrary) curvature, and we will investigate the role and evolution of the curvature during the particle creation processes. The feedback of the created matter on the cosmological expansion will also be considered.

The present paper is organized as follows. The basic formalism of the irre-

versible thermodynamics of open systems, together with the principles of warm inflation, are reviewed in Section 2. The full set of equations describing warm inflationary models with irreversible radiation and dark matter production due to the decay of the scalar field are written down in Section 3. As an application of the considered warm inflationary scenario in Section 4 we consider the case of the coherently oscillating scalar field, which is analyzed by numerically integrating the system of the gravitational field equations. We discuss and conclude our results in Section 5.

2. THERMODYNAMICS AND WARM INFLATION

In the present Section we will briefly review the fundamentals of the thermodynamics of open systems in the presence of particle creation, and we will also present the standard formulation of the warm inflationary cosmological scenario.

2.1. BRIEF REVIEW OF IRREVERSIBLE THERMODYNAMICS WITH MATTER CREATION

The basic equations describing the thermodynamic of irreversible processes in the presence of matter creation have been developed in Prigogine *et al.* (1988). In particular, the time variation of the matter energy density in systems with particle production is given by

$$\dot{\rho} = \frac{h}{n} \dot{n}, \quad (1)$$

where the overdot designates the derivative with respect to time, h is the enthalpy per unit volume, defined as $h = \rho + p$, where ρ and p denote the energy density and the thermodynamic pressure, respectively. Another important concept in the theory of irreversible processes with matter creation is the creation pressure, defined as (Prigogine *et al.*, 1988),

$$p_c = -\frac{h}{n} \frac{d(nV)}{dV} = -\frac{h}{n} \frac{V}{\dot{V}} \left(\dot{n} + \frac{\dot{V}}{V} n \right), \quad (2)$$

where V is the comoving volume, and n denotes the particle number density, defined as $n = N/V$. The introduction of the creation pressure in the thermodynamical formalism represents an effective way to describe particle production, allowing the reformulation of the second law of thermodynamics for adiabatic systems in the presence of matter creation as (Prigogine *et al.*, 1988),

$$d(\rho V) + (p + p_c) dV = 0. \quad (3)$$

In the following we will assume that the Universe can be described by a homogenous and isotropic geometry, given by the Friedmann-Lemaître-Robertson-Walker

(FLRW) metric, which has the general form

$$ds^2 = c^2 dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (4)$$

where $k = 0, \pm 1$ represents the curvature of the space. Dimensionally, k has units of length^{-2} , r has units of length, and $a(t)$ is dimensionless. An important cosmological quantity is the Hubble function, defined as $H(t) = \dot{a}(t)/a(t)$. For the FLRW metric, the comoving volume is given by $V(t) = a^3(t)$, giving $\dot{V}(t) = 3a^2(t)\dot{a}(t)$. Therefore, for the creation pressure we obtain

$$p_c = -\frac{\rho + p}{n} \frac{a}{3\dot{a}} \left(\dot{n} + \frac{3\dot{a}}{a} n \right) = -\frac{\rho + p}{3nH} (\dot{n} + 3Hn). \quad (5)$$

In order to close the thermodynamic equations we need a supplementary relation for the variation of the particle number, which in a cosmological framework can generally be taken as

$$\dot{n} + 3Hn = \Psi, \quad (6)$$

where Ψ is a source term. When $\Psi > 0$, particle creation occurs, while $\Psi < 0$ corresponds to matter/particle decay. Hence, the creation pressure is obtained as

$$p_c = -\frac{\rho + p}{3nH} \Psi. \quad (7)$$

2.2. STANDARD FORMULATION OF WARM INFLATION

The warm inflationary scenario is an interesting and important theoretical alternative to the cold inflation and reheating theories (Berera and Fang, 1995; Berera, 1995). In the warm inflation scenario, the Universe experiences an accelerated very early expansionary stage, triggered by the presence of a scalar field. But, as opposed to the cold inflation model, besides the scalar field, a matter component of the cosmological fluid, usually assumed to be radiation, is also present, being generated by the decay of the scalar field. During the cosmological evolution, these two components interact dynamically. In its standard formulation, the cosmological evolution in warm inflation is described by the Friedmann equations

$$3H^2 = \frac{1}{M_P^2} (\rho_\phi + \rho_{rad}), \quad (8)$$

and

$$2\dot{H} = -\frac{1}{M_P^2} \left(\dot{\phi}^2 + \frac{4}{3} \rho_{rad} \right), \quad (9)$$

respectively, where by $M_P = \sqrt{\hbar c/G}$ we have denoted the Planck mass, ϕ is the scalar field, and ρ_ϕ and ρ_{rad} represents the energy densities of the scalar field and

radiation, respectively. The energy density ρ_ϕ and the pressure p_ϕ of the scalar field are given by

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi), \quad (10)$$

and

$$p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi), \quad (11)$$

respectively, where $V(\phi)$ is the self-interaction potential of the field.

Due to the decay of the scalar field, which is essentially a dissipative process, energy is transferred from the field to the radiation fluid. In the standard warm inflationary scenario this process is described by the following energy balance equations (Berera and Fang, 1995; Berera, 1995),

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = -\Gamma\dot{\phi}^2, \quad (12)$$

$$\dot{\rho}_{rad} + 3H(\rho_{rad} + p_{rad}) = \Gamma\dot{\phi}^2, \quad (13)$$

where $\Gamma > 0$ is the dissipation coefficient. By taking into account the explicit expressions of the scalar field energy density and pressure, Eq. (12) takes the form of a Klein-Gordon type equation,

$$\ddot{\phi} + 3H(1 + Q)\dot{\phi} + V'(\phi) = 0, \quad (14)$$

where $Q = \Gamma/3H$. In the following we will consider the natural system of units with $\hbar = G = 1$, and $M_P^2 = 1$, respectively.

3. PARTICLE CREATION IN THE WARM INFLATIONARY SCENARIO IN A CURVED UNIVERSE

In the following we consider a curved Universe, described by the general FLRW metric (4), and containing three basic components: a decaying scalar field, which generates both radiation and a (dark) matter component. We denote the particle number densities of these components as n_ϕ , n_{rad} , and n_{DM} , respectively. The radiation fluid satisfies the equation of state $p_{rad} = \rho_{rad}/3$, with $\rho_{rad} = 8\pi^5 T^4/15$, where T is the temperature of the fluid. The photon number density is given as a function of the temperature by $n_{rad} = 16\pi\zeta(3)T^3$, where $\zeta(n)$ is the Riemann zeta function. We also assume that dark matter is produced with a negligible pressure, $p_{DM} = 0$, and therefore $\rho_{DM} = m_{DM}n_{DM}$, where m_{DM} is the mass of the dark matter particle.

In order to apply the formalism of the thermodynamics of open systems, we need to formulate first the particle balance equations, which originate from Eq. (6). Moreover, in the following we assume that the source term Ψ is proportional to the energy density ρ_ϕ of the scalar field, the proportionality coefficient being given by

the dissipation coefficient Γ , also giving the particle creation/decay rates. Hence, the balance equations of the particle number densities take the following form,

$$\dot{n}_\phi + 3Hn_\phi = -\Gamma \frac{\rho_\phi}{m_\phi}, \quad (15)$$

$$\dot{n}_{rad} + 3Hn_{rad} = \frac{\Gamma}{2} \frac{\rho_\phi}{m_\phi}, \quad (16)$$

$$\dot{n}_{DM} + 3Hn_{DM} = \frac{\Gamma}{2} \frac{\rho_\phi}{m_\phi}, \quad (17)$$

where m_ϕ is the mass of the scalar field particle, and the dissipation coefficients have been chosen in such a way that the total particle number $n = n_\phi + n_{rad} + n_{DM}$ is conserved. Once the particle balance equations are known, from Eq. (1) one obtains the energy densities of the newly created particles as

$$\dot{\rho}_\phi = (\rho_\phi + p_\phi) \frac{\dot{n}_\phi}{n_\phi} = \dot{\phi}^2 \left(-3H - \Gamma \frac{\rho_\phi}{m_\phi n_\phi} \right), \quad (18)$$

$$\dot{\rho}_{rad} = (\rho_{rad} + p_{rad}) \frac{\dot{n}_{rad}}{n_{rad}} = \frac{4}{3} \rho_{rad} \left(-3H + \frac{\Gamma}{2} \frac{\rho_\phi}{m_\phi n_{rad}} \right), \quad (19)$$

$$\dot{\rho}_{DM} = (\rho_{DM} + p_{DM}) \frac{\dot{n}_{DM}}{n_{DM}} = \rho_{DM} \left(-3H + \frac{\Gamma}{2} \frac{\rho_\phi}{m_\phi n_{DM}} \right). \quad (20)$$

For the creation pressures, given by Eq. (7), we obtain

$$p_c^{(\phi)} = \Gamma \frac{\dot{\phi}^2}{3H} \frac{\rho_\phi}{m_\phi n_\phi}, \quad (21)$$

$$p_c^{(rad)} = -\frac{2\Gamma}{3} \frac{\rho_{rad}}{3n_{rad}H} \frac{\rho_\phi}{m_\phi}, \quad (22)$$

$$p_c^{(DM)} = -\frac{\Gamma}{2} \frac{\rho_{DM}}{3n_{DM}H} \frac{\rho_\phi}{m_\phi}. \quad (23)$$

The basic equations describing the dynamical evolution of the scale factor in the warm inflationary curved Friedmann-Lemaitre-Robertson-Walker Universe filled with a decaying scalar field, and radiation and dark matter creation, respectively, are given by

$$3\frac{\dot{a}^2}{a^2} + 3\frac{k}{a^2} = \frac{1}{M_P^2} (\rho_\phi + \rho_{rad} + \rho_{DM}), \quad (24)$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = -\frac{1}{M_P^2} \left[p_\phi + p_c^{(\phi)} + p_{rad} + p_c^{(rad)} + p_{DM} + p_c^{(DM)} \right], \quad (25)$$

respectively. In the following we take $M_P = 1$, according to $\hbar = G = 1$. The equation for the radiation particle number can be immediately converted into an equation

giving the variation of the temperature of the Universe as

$$\dot{T} + HT = \frac{\Gamma}{96\pi\zeta(3)} \frac{1}{T^2} \frac{\rho_\phi}{m_\phi}, \quad (26)$$

while the radiation creation pressure becomes

$$p_c^{(rad)} = -\frac{\Gamma\pi^4}{135\zeta(3)} \frac{T}{H} \frac{\rho_\phi}{m_\phi}. \quad (27)$$

Equation (18) for the energy density variation of the scalar field takes the form of a generalized Klein-Gordon equation, given by

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = -\Gamma\dot{\phi} \frac{\rho_\phi}{m_\phi n_\phi}. \quad (28)$$

4. WARM INFLATION IN THE PRESENCE OF A COHERENT SCALAR FIELD

In the following we will investigate a simple warm inflationary cosmological model, obtained under the assumption that the inflationary scalar field is a homogeneous field, oscillating with a frequency m_ϕ . Such a field can be interpreted as a coherent wave of scalar “particles” having zero momenta, and with the corresponding particle number density given by

$$n_\phi = \frac{\rho_\phi}{m_\phi}. \quad (29)$$

Therefore, this model corresponds to n_ϕ oscillators, having the same frequency m_ϕ , with all scalar particles oscillating coherently with the same phase. Hence, the warm inflationary evolution can be described as determined by a single homogeneous scalar wave $\phi(t)$. For this type of scalar field it follows that $p_\phi = 0$, giving

$$V(\phi) = \frac{1}{2}\dot{\phi}^2, \quad (30)$$

and

$$\rho_\phi = \dot{\phi}^2, \quad (31)$$

respectively.

Under this choice of the scalar field, the set of cosmological equations describing the warm inflationary dynamics takes the form

$$\dot{\rho}_\phi + 3H\rho_\phi = -\Gamma\rho_\phi, \quad (32)$$

$$\dot{T} + HT = \frac{\Gamma}{96\pi\zeta(3)} \frac{1}{m_\phi T^2} \rho_\phi, \quad (33)$$

$$\dot{\rho}_{DM} + 3H\rho_{DM} = \frac{\Gamma}{2} \frac{m_{DM}}{m_\phi} \rho_\phi, \quad (34)$$

$$3\frac{\dot{a}^2}{a^2} + 3\frac{k}{a^2} = \frac{1}{M_P^2} \left(\rho_\phi + \frac{8\pi^5}{15} T^4 + \rho_{DM} \right). \quad (35)$$

The system of differential equations (32)-(35) represents a system of four ordinary differential equations with four unknowns $\{a, \rho_\phi, \rho_{DM}, T\}$. It must be integrated with the initial conditions $a(0) = a_0$, $\rho_\phi(0) = \rho_{\phi 0}$, $\rho_{DM}(0) = \rho_{DM 0}$, $T(0) = T_0$.

In order to simplify the mathematical formalism we introduce a set of dimensionless variables $\{\tau, A, r_\phi, r_{DM}, \theta\}$, defined as

$$\begin{aligned} \tau &= \Gamma t, \quad A = \Gamma a, \quad r_\phi = \frac{1}{3M_P^2 \Gamma^2} \rho_\phi, \\ r_{DM} &= \frac{1}{3M_P^2 \Gamma^2} \rho_{DM}, \quad \theta = \left(\frac{M_P^2 \Gamma^2}{32\pi\zeta(3)m_\phi} \right)^{-1/3} T. \end{aligned} \quad (36)$$

Then the system of equations (32)-(35) takes the following dimensionless form,

$$\frac{dr_\phi}{d\tau} + 3\frac{1}{A} \frac{dA}{d\tau} r_\phi = -r_\phi, \quad (37)$$

$$\frac{d\theta}{d\tau} + \frac{1}{A} \frac{dA}{d\tau} \theta = \frac{r_\phi}{\theta^2}, \quad (38)$$

$$\frac{dr_{DM}}{d\tau} + 3\frac{1}{A} \frac{dA}{d\tau} r_{DM} = \frac{\lambda}{2} r_\phi, \quad (39)$$

$$\frac{1}{A^2} \left(\frac{dA}{d\tau} \right)^2 + \frac{k}{A^2} = r_\phi + \alpha \theta^4 + r_{DM}, \quad (40)$$

where we have denoted

$$\lambda = \frac{m_{DM}}{m_\phi}, \quad \alpha = \frac{8\pi^5}{45M_P^2 \Gamma^2} \left(\frac{M_P^2 \Gamma^2}{32\pi\zeta(3)m_\phi} \right)^{4/3}. \quad (41)$$

We have chosen the numerical values for the initial conditions used for the numerical integration of the cosmological evolution equations: $A_0 = 0.1$, $\theta_0 = 0.0001$, $r_{DM 0} = 0.005$, $r_{\phi 0} = 20$. For the dimensionless parameters of the model we adopt the values $\lambda = 6$ and $\alpha = 10$, whereas for the geometry of the primordial Universe we successively consider all three possibilities – flat, open and closed, respectively.

The variation of the energy density of the scalar field, and of the temperature of the radiation fluid are represented in Figure 1. As one can see from the left panel of the figure, the scalar field decays rapidly due to matter creation, but there is no dependence on the initial geometry of the Universe, therefore all cases have a very similar decay. In the temperature evolution of the radiation fluid (right panel of Figure 1) a dependence can be seen between the maximum reheating temperature and the initial geometry of the Universe. The reheating temperature, which is the maximum

temperature of the Universe acquired by the end of inflation, is slightly reduced by a shift from an initially flat Universe to an open one and slightly increased considering a closed initial Universe, which moreover seems to contain a remnant component in a later Universe.

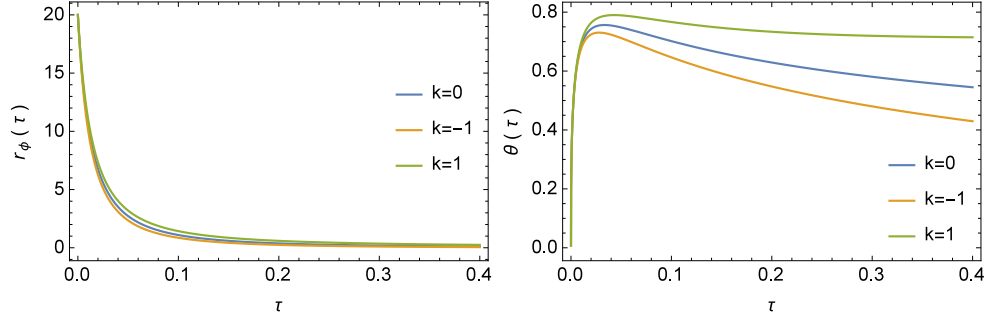


Fig. 1 – Time variation in a warm inflationary regime of the energy density of the scalar field $r_\phi(\tau)$ (left panel) and of the temperature $\theta(\tau)$ (right panel) in a Universe having an initial flat geometry, $k = 0$ (blue curve), an open geometry, $k = -1$ (orange curve), and a closed geometry, $k = 1$ (green curve), respectively.

The same dependence on the initial curvature of the Universe can be observed in the time variation of the energy density of the newly created dark matter, presented in Figure 2, with the maximum values of the dark matter density occurring at roughly similar times as compared to the maximum values of the temperature of the radiation fluid. Analogous to the temperature function, the closed geometry also shows some remnants of dark matter particles produced during the warm inflationary era.

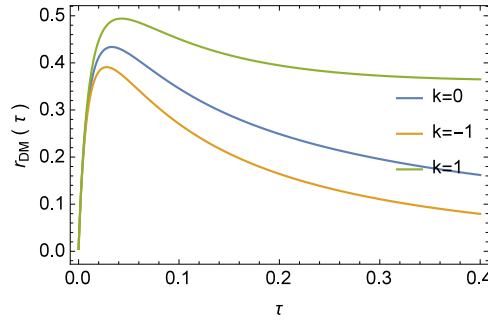


Fig. 2 – Time variation in a warm inflationary regime of the dark matter energy density $r_{DM}(\tau)$ in a Universe having an initial flat geometry, $k = 0$ (blue curve), an open geometry, $k = -1$ (orange curve), and a closed geometry, $k = 1$ (green curve), respectively.

The time variations of the scale factor and of the curvature terms $|k|/A^2(\tau)$ are depicted in Figure 3. The expansion rate also depends on the initial curvature, with

the slowest expansion corresponding to the closed cosmological geometry. While the curvature terms reach quickly the zero value in the open and flat cases, in the present model this is not the case for an initial closed Universe, whose curvature may reach the zero value only in the asymptotic limit of very large times.

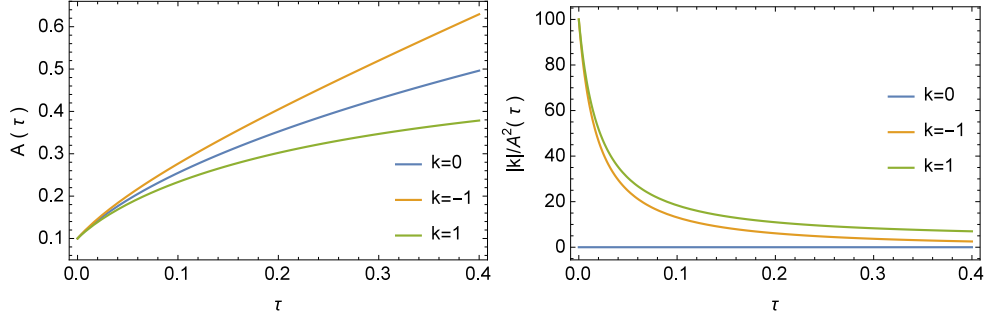


Fig. 3 – Time variation in a warm inflationary regime of the scale factor $A(\tau)$ (left panel), and of the curvature term $|k|/A^2(\tau)$ (right panel) in a Universe having a flat geometry, $k = 0$ (blue curve), an open geometry, $k = -1$ (orange curve), and a closed geometry, $k = 1$ (green curve), respectively.

5. DISCUSSIONS AND FINAL REMARKS

The present paper aims to address from an open thermodynamical system perspective the problem of the creation of particles in a warm inflationary scenario by considering a three-component dynamical system composed of a scalar field, radiation, and dark matter, respectively. Moreover, we have assumed that the Universe may have had an initial curvature at the moment of its very beginning, and we have explored the role this curvature may have had on the evolution of the physical properties of the cosmological system. The work conducted in this paper intends to enlarge the approach of Harko and Sheikhahmadi (2020), by including a dark matter and a curvature component into the cosmological field equations of the warm inflationary formalism.

In our study we have considered the early Universe as an open thermodynamic systems in which entropy and particle creation occurs (Prigogine *et al.*, 1988). The time evolution of the dynamically interacting cosmological fluid consisting of a scalar field, radiation and dark matter has been investigated in the curved FLRW geometry, with the effects of the geometric curvature terms fully taken into account.

Some important features of this model can be observed from the behaviour of the physical and geometrical parameters, as obtained in the previous Section. As the scalar field decays into the newly created radiation and dark matter particles (see Figure 1), the temperature of the early Universe is bound to increase, in the case of a flat, $k = 0$, open, $k = -1$, or closed, $k = 1$, geometry, from a zero value to a

maximum value. After reaching its maximum, the temperature decreases, due to the accelerated expansion of the Universe.

The maximum value of the photon gas temperature, which can be considered as the reheating temperature, is an important cosmological parameter. In order to obtain the reheating temperature values, we mention first that the evolution equation of the scalar field energy density, Eq. (37), can be integrated exactly to give the expression

$$r_\phi = r_{\phi 0} \frac{e^{-\tau}}{A^3}, \quad (42)$$

where $r_{\phi 0}$ is an arbitrary constant of integration. From Eq. (38) we obtain the maximum value of the temperature as

$$\theta_{max} = \left[\frac{r_\phi}{(1/A)(dA/d\tau)} \right]^{1/3} \Big|_{\tau=\tau_{max}}. \quad (43)$$

From Figure 1 one can obtain the reheating maximum temperatures θ_{max} , as $\theta_{max}^{(-1)}(\tau_{max} = 0.02693) = 0.7258$ for the open geometry, $\theta_{max}^{(0)}(\tau_{max} = 0.03293) = 0.7585$ for the flat geometry, and $\theta_{max}^{(+1)}(\tau_{max} = 0.04133) = 0.7912$ for the closed geometry.

The energy density of the dark matter particles $r_{DM}(\tau)$ has a similar behavior to that of the temperature. The maximum value of the dark matter energy density can be obtained from Eq. (39) as,

$$r_{DM}^{(max)} = \frac{\lambda r_\phi}{6(1/A)(dA/d\tau)} \Big|_{\tau=\tau_{max}}. \quad (44)$$

For an open Universe, $r_{DM}^{(max)} \Big|_{k=-1}(\tau_{max} = 0.02767) = 0.3872$, for a flat Universe $r_{DM}^{(max)} \Big|_{k=0}(\tau_{max} = 0.03257) = 0.4316$ and for a closed Universe, $r_{DM}^{(max)} \Big|_{k=1}(\tau_{max} = 0.04237) = 0.4916$. It is interesting to note that a decrease in λ reduces the peak value of the dark matter density, so that for a value of $\lambda_{critic} = 0.1$, the maximum disappears. Taking into consideration the definition of λ , a relationship between the mass of the dark matter particles and the mass of the scalar field particle can be found as $\lambda_{critic} = 0.1 = m_{DM}/m_\phi$, and therefore $m_{DM} \geq 0.1 m_\phi$.

We have also investigated, in our analysis, the behavior of the curvature terms during the warm inflationary period. For an open Universe, the curvature term $|k|/A^2(\tau)$ heads towards the zero value, indicating that the Universe reaches the flat geometry in a very short time interval. Nevertheless, in a closed geometry, the curvature term does not approach zero, which implies the necessary existence of a remnant curvature in the late Universe. Moreover, the closed geometry scenario imposes a strong requirement for the initial conditions of the Universe, so that real

solutions of the field equations can be found, namely,

$$\frac{1}{3} (r_\phi + \alpha\theta^4 + r_{DM})|_{\tau=\tau_0} > 1. \quad (45)$$

For zero initial values of the temperature of the radiation fluid, and for the density of the dark matter, it follows that $r_\phi|_{\tau=\tau_0}$ must be strictly greater than 3. Hence, an initially closed Universe can not exist, unless there is a fine-tuning with regard of the initial conditions.

According to the standard Big Bang model, the reheating regime must end at the reheating time $t_{reh} = 10^{-18}$ s, which signifies the time at which the reheating temperature is attained, with the corresponding reheating temperature being of the order of $T_{reh} < 10^{15}$ GeV. Therefore, Γ can be computed from the maximum of τ , its value depending directly upon the geometry of the early Universe, $\Gamma = \tau_{max}^{(k=\{0;\pm 1\})}/t_{reh}$.

Moreover, we can aquire some information about the mass of the dark matter particles considering Eq. (36) where we substitute Γ with the previously presented form, from where

$$m_{DM} = \lambda \frac{M_P^2}{32\pi\zeta(3)} \frac{\tau_{max}^2}{t_{reh}^2} \frac{\theta_{max}^3}{T_{reh}^3}. \quad (46)$$

Therefore, the dark matter particle mass has the following numerical value at the end of the reheating regime:

$$m_{DM} = 5.34 \times 10^{-22} \times \lambda \times \tau_{max}^2 \times \theta_{max}^3 \times \left(\frac{t_{reh}}{10^{-18} \text{ s}}\right)^{-2} \times \left(\frac{T_{reh}}{10^{15} \text{ GeV}}\right)^{-3} \text{ GeV}. \quad (47)$$

If we choose for the mass coefficient the value $\lambda = 6$ and for the maximum of the photon gas temperature the value $\theta_{max} = 0.759$, its peak occurring at the time $\tau_{max} = 0.034$, we obtain for the dark matter mass the value $m_{DM} = 1.62 \times 10^{-15}$ eV. Moreover, by the same numerical selection of λ , θ_{max} and τ_{max} , yet increasing the reheating temperature by two orders of magnitude, the mass reaches 10^{-21} eV. This result is comparable to the mass of an ultralight axion-like particle measured by gravitational weak lensing in the study of Dentler *et al.* (2022), in which the dark matter particle is determined to have a mass of the order of 10^{-22} eV.

From the analysis of the static Bose-Einstein condensate dark matter halos it follows that the condensate dark matter satisfies a mass-galactic radius relation of the form (Böhmer and T. Harko, 2007)

$$m = \left(\frac{\pi^2 \hbar^2 a}{GR^2}\right)^{1/3} \approx 6.73 \times 10^{-2} \times [a(\text{fm})]^{1/3} [R(\text{kpc})]^{-2/3} \text{ eV}, \quad (48)$$

where a is the scattering length of the particle, and R is the galactic radius. By introducing the ratio $\sigma_m = \sigma/m$ of the self-interaction cross section $\sigma = 4\pi a^2$, and

of the dark matter particle mass m , then the mass of the dark matter particle can be obtained from Eq. (48) as Harko *et al.* (2015)

$$m = \left(\frac{\pi^{3/2} \hbar^2}{2G} \frac{\sqrt{\sigma_m}}{R^2} \right)^{2/5}. \quad (49)$$

By assuming for σ_m a value of the order of $\sigma_m = 1.25 \text{ cm}^2/\text{g}$ (Harko *et al.*, 2015), we obtain for the mass of the dark matter particle the constraint upper limit of the order

$$\begin{aligned} m &< 3.1933 \times 10^{-37} \left(\frac{R}{10 \text{ kpc}} \right)^{-4/5} \times \left(\frac{\sigma_m}{1.25 \text{ cm}^2/\text{g}} \right)^{1/5} \text{ g} \\ &= 0.1791 \times \left(\frac{R}{10 \text{ kpc}} \right)^{-4/5} \times \left(\frac{\sigma_m}{1.25 \text{ cm}^2/\text{g}} \right)^{1/5} \text{ meV}. \end{aligned} \quad (50)$$

By assuming that the self-interaction cross section of the dark matter particle takes values in the range $\sigma_m \in (0.00335 \text{ cm}^2/\text{g}, 0.0559 \text{ cm}^2/\text{g})$, we obtain for the possible range of the mass of the dark matter as given by

$$\begin{aligned} m &\approx (9.516 \times 10^{-38} - 1.670 \times 10^{-37}) \left(\frac{R}{10 \text{ kpc}} \right)^{-4/5} \text{ g} \\ &= (0.053 - 0.093) \left(\frac{R}{10 \text{ kpc}} \right)^{-4/5} \text{ meV}. \end{aligned} \quad (51)$$

In order to find the dark matter mass m_{DM} in the upper limit outlined in Harko *et al.* (2015) in the form of Eq. (51), the parameters λ , t_{reh} , T_{reh} from Eq. (46) must be adjusted accordingly. A three-dimensional representation of the dark matter mass as a function of the mass coefficient and reheating time is illustrated in Figure 4. One can clearly see the proportionality of m_{DM} to the mass coefficient, as the dark matter mass grows with the increase of λ . Moreover, there is an inverse proportionality of m_{DM} to the reheating time and temperature, so that although the system accumulates mass, which is of the order of 10^{-15} eV , the reheating time and temperature diminish in value.

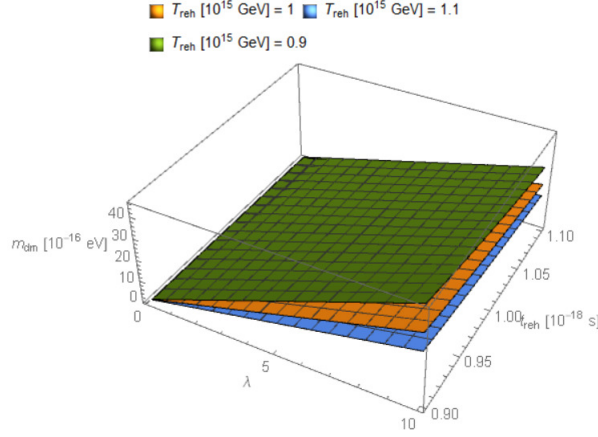


Fig. 4 – Dark matter mass variation as a function of mass coefficient and reheating time when $t_{reh} = 10^{-18}$ s and $T_{reh} = 10^{15}$ GeV (orange plane), when there is a 10 percent increase of the reheating temperature value (blue plane), and at a 10 percent decrease of the reheating temperature value, respectively (green plane).

The representation of the dark matter mass as a function of λ and of the photon gas temperature can be seen in Figure 5. It is notable that by increasing the reheating temperature from 0.9×10^{15} GeV to 2.0×10^{15} GeV, the mass function reveals a decreasing behaviour at a steep rate for high enough mass coefficients λ . The dark matter mass enlarges as the reheating time is decreased along with the temperature by ten percent of their original value.

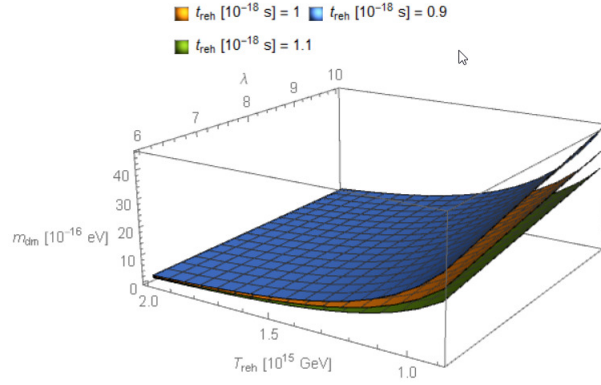


Fig. 5 – Dark matter mass variation as a function of mass coefficient and reheating temperature when $t_{reh} = 10^{-18}$ s and $T_{reh} = 10^{15}$ GeV (orange plane), when there is a 10 percent decrease of the reheating time value (blue plane), and at a 10 percent increase of the reheating time value, respectively (green plane).

The present paper has only considered a warm inflationary scenario in the presence of a homogeneous coherently oscillating scalar field, which has given a simple cosmological model with an interacting three-components cosmological fluid, whose evolution is described by using the thermodynamics of open systems in the presence of matter creation. Additional forms of the scalar field will be taken into consideration in a future study.

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