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VISCOUS MODIFIED GHOST SCALAR FIELD DARK ENERGY MODELS WITH VARYING G

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Abstract. We study QCD-modified scalar field models of dark energy, in the presence of both interaction and viscosity, with varying gravitational constant G. The equation of the state parameter of the interacting viscous QCD-modified ghost dark energy (MGDE) and the deceleration parameter of the universe, are derived. Furthermore, we establish the correspondence between the interacting viscous QCD-MGDE and scalar field models of the dark energy which includes quintessence, tachyon, k-essence, and dilaton energy density. This is done in the framework of a non-flat FRW universe. Hence we are able to establish a correspondence and reconstruct the potential $V(\phi)$ and dynamics ϕ of the scalar field models according to the evolution of viscous QCD-MGDE.

Key words: FRW universe, modified ghost dark energy, interacting viscous modified ghost dark energy, scalar field dark energy.

1. INTRODUCTION

Observational data from the Type Ia supernovae suggests that the universe is dominated by the two dark components, dark matter and dark energy (Perlmutter *et al.*, 1997, 1998, 1999; Riess *et al.*, 2004, 2007). Dark matter, which is described as matter without pressure, shows its presence through galactic rotation curves and large-scale structure formation, whereas dark energy, better known as an exotic energy with negative pressure, describes the present cosmic accelerating expansion. Several mechanisms are put forth to describe the physical characteristics of this dark energy. The simplest theoretical candidate for such a component is a positive cosmological constant, but it has drawback associated with the cosmic coincidence problems and fine tuning as well (Copeland and Sheykhi, 2006; Weinberg, 1989).

In the aspect of Veneziano ghost of quantum chromodynamics (QCD), a dynamical DE known as Veneziano ghost DE has been described. The Veneziano ghost field plays a major role in the resolution of the $U(1)_A$ problem in QCD (Kawarabayashi and Ohta, 1980; Nath and Arnowitt, 1981; Rosenzweig *et al.*, 1980; Veneziano, 1979;

Witten, 1979). The ghost field does not contribute to the vacuum energy density in Minkowski space-time, whereas in curved space-time, it gives rise to a vacuum energy density proportional to $\Lambda^3_{QCD}H$ (Holdom, 2011; Thomas and Zhitnitsky, 2012; Urban and Zhitnitsky, 2010; Zhitnitsky, 2010). Here, Λ_{QCD} is QCD mass scale and H is Hubble parameter. This small contribution can play a vital role in the evolutionary behavior of the universe. For instance, considering $H \sim 10^{-33}_{e}V$ at the present, $\Lambda^3_{QCD}H$ gives the correct order of observed magnitude of the DE density. This coincidence is remarkable and indicates that the GDE model gets rid of fine tuning problem (Ohta, 2011; Urban and Zhitnitsky, 2009, 2010). In addition, the appearance of the QCD scale could be useful for a solution to the cosmic coincidence problem, as it may be the scale at which the formation of dark matter takes place (Forbes and Zhitnitsky, 2008). GDE model has an advantage when compared other DE models. It is that the DE can be completely explained within the standard model and GR, without resorting to any new degree(s) of freedom, new field, new symmetries or modifications to general relativity (Cai *et al.*, 2011, 2012; Ohta, 2011; Urban and Zhitnitsky, 2009, 2010).

The ordinary GDE density is given by (Ohta, 2011; Urban and Zhitnitsky, 2009, 2010) $\rho_D = \alpha H$, where α is a constant with dimension $[energy]^3$, and roughly of order of $\Lambda^3_Q CD$. This GDE model has got a lot of interest recently in the literature (Cai *et al.*, 2011; Ebrahimi and Sheykhi, 2011a,b; Khodam *et al.*, 2012; Rozas, 2012; Sheykhi and Bagheri, 2011; Sheykhi and Movahed, 2012a; Sheykhi *et al.*, 2012b). Although the ordinary GDE model is consistent with the observational data, it fails to describe the early evolution of the universe. With this motivation, (Cai *et al.*, 2012) introduced the modified GDE density as

$$\rho_D = \alpha H + \beta H^2, \tag{1}$$

where the constant α is same as that described in the ordinary GDE and β is another constant with dimension $[energy]^2$.

Veneziano ghost field in QCD, finds its origin in the vacuum energy of quantum fields which is expressed as $H + \mathcal{O}(H^2)$ (Zhitnitsky, 2012). However, in the ordinary GDE model, only the leading term H has been considered. The subleading term H^2 is too small and cannot lead to the accelerating expansion of the universe. Cai *et al.* (2012), however showed that this term can play an important role in the early evolution of the universe, acting as an early DE. Cai *et al.* (2012) used the joint analysis of the astronomical data, and concluded that the subleading term H^2 of the DE density, which is the early DE, could exhibit a fraction energy density around 10%. Indeed, the term proportional to H^2 finds its relation to the difference between the vacuum energies in Minkowski spacetime and in a FRW universe (Maggiore, 2011a; Maggiore *et al.*, 2011b). On the other hand, the difference in vacuum energy arising from the Veneziano ghost field, is introduced, in order to solve the so-called

$U(1)_A$ problem in QCD. It has the exact form $\alpha H + \beta H^2$, where $\alpha \sim \Lambda^3_{QCD} \sim (100\ \mathrm{MeV})^3$.

It is known that the scalar field models (such as quintessence, K-essence, tachyon, and dilaton) are effective in describing an underlying theory of DE (Wu *et al.*, 2008). Scalar fields arise naturally in particle physics, which include super symmetric field theories, as well as string/M theory. Hence, scalar field is expected to reveal the nature of DE, as well as its dynamical mechanism. The dynamics of these scalar field models and the corresponding potential have been widely investigated in the scenario of different DE models (Granda and Oliveros, 2009; Jamil and Farooq, 2010; Jawad and Majeed, 2015; Karami and Fehri, 2010; Rozas *et al.*, 2010; Rozas, 2011; Setare, 2007; Zhang, 2007; Zhang *et al.*, 2007). Moreover, Karami and Abdolmaleki (2012); Karami *et al.* (2013) have discussed the dynamics of scalar field and corresponding potential in the presence of viscous ghost as well as generalized ghost DE models in non-flat universe. Karami *et al.* (2013) studied the Veneziano MGDE in the framework of non-flat FRW universe filled with interacting viscous MGDE and DM and then obtained the EoS parameter of the interacting viscous MGDE and deceleration parameter q. Furthermore, they reconstructed both dynamics and potentials of the quintessence, tachyon, k-essense and DE models of dilaton scalar field, in accordance with the evolution of the MGDE density. The authors also studied the effects of viscous parameter on the dynamics of scalar field, corresponding potential, and derived interesting results.

In this paper, we generalize the work of Karami et al. Karami *et al.* (2013) in the framework of FRW cosmology and study the QCD modified ghost scalar field model of the dark energy in the presence of both interacting and viscosity with time varying gravitational constant G. Furthermore, by using a correspondence between interacting viscous MGDE with various scalar field models of DE, we reconstruct the dynamics ϕ and potential $V(\phi)$ of the scalar field models according to the evolution of the MGDE density.

This paper is organized as follows: in section 2, we study interacting viscous modified ghost dark energy model with time varying gravitational constant G in the framework of non-flat FRW cosmology. We obtain the EoS parameter, deceleration parameter and the differential equation of MGDE density parameter for the interacting viscous modified ghost dark energy model in a non-flat universe. In section 3, we establish a correspondence between the interacting viscous MGDE with time varying gravitational constant G and the quintessence, tachyon, k-essense and dilaton scalar field models of dark energy. Furthermore, we reconstruct the dynamics ϕ and potentials $V(\phi)$ for these scalar field models of DE according to the evolution of MGDE density, which describe accelerated expansion of the universe. The last section 4, is devoted to the conclusions.

2. FRW MODEL WITH VISCOUS MODIFIED GHOST DARK ENERGY

We consider, the spatially homogeneous and isotropic Friedmann-Robertson-Walker (FRW) space-time the line element given by Weinberg (1972)

$$ds^2 = -dt^2 + a^2(t)\left[\frac{1}{1-kr^2}dr^2 + r^2\left(d\theta^2 + sin^2\theta d\phi^2\right)\right], \tag{2}$$

where $a(t)$ is a scale factor of the universe with cosmic time t. The θ and ϕ parameters are the usual azimuthal and polar angles of spherical coordinates, with $0 \le \theta \le \pi$ and $0 \le \phi \le 2\pi$. The coordinates r, θ and ϕ are known as co-moving coordinates. The constant k is the curvature parameter, which describes the geometry of the spatial section of space-time with closed, flat and open space-times corresponding to $k = 1,\ 0,\ -1$ respectively.

The evolution equations for the scale factor and matter density follows from Einstein's field equations (Weinberg, 1972)

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}, \tag{3}$$

where $G_{\mu\nu}$ is the Einstein tensor, $R_{\mu\nu}$ is the Ricci tensor, $g_{\mu\nu}$ is the metric tensor, R is the Ricci scalar, $T_{\mu\nu}$ is the energy momentum tensor and the varying gravitational constant G is G_{x^μ}.

The presence of bulk viscosity ζ in cosmic fluids, give rise to the dissipative effects in an isotropic and homogeneous FRW universe.

The energy momentum tensor for a relativistic fluid with bulk viscosity is given by

$$T_{\mu\nu} = (\rho + \bar{p})\,U_\mu U_\nu + \bar{p}g_{\mu\nu}, \tag{4}$$

where ρ denotes the energy density, p is thermodynamic pressure, and $\bar{p}$ is the effective thermodynamic pressure respectively.

The effective thermodynamic pressure $\bar{p}$, is given by

$$\bar{p} = p + \Pi, \tag{5}$$

where Π is the bulk viscous pressure.

Therefore, the first Friedmann Equation is given by

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\left(\rho_D + \rho_m\right), \tag{6}$$

where k is the curvature parameter, ρ_D and ρ_m are energy densities of MGDE and DM respectively.

The dimensionless density parameters are defined as

$$\Omega_m = \frac{8\pi G \rho_m}{3H^2}, \tag{7}$$

$$\Omega_D = \frac{8\pi G \rho_D}{3H^2}, \tag{8}$$

$$\Omega_k = \frac{k}{a^2 H^2}. \tag{9}$$

Using the density parameters given in Eqs. (7) to (9), Friedmann Equation (Eq. 6) can be rewritten as

$$1 + \Omega_k = \Omega_D + \Omega_m. \tag{10}$$

Substituting Eq. (1) in Eq. (8), we obtain the expression for the density parameter Ω_D as

$$\Omega_D = \frac{8\pi G \alpha}{3H} + 1 - \gamma, \tag{11}$$

where for convenience, the parameter γ is defined by (Cai *et al.*, 2012).

$$\gamma = 1 - \frac{8\pi G \beta}{3}. \tag{12}$$

Using Eq. (11), the curvature energy density parameter in Eq. (9), can be written as

$$\Omega_k = \frac{\Omega_{k_0}}{(\Omega_{D_0} + \gamma - 1)^2} \left(\frac{\Omega_D + \gamma - 1}{a} \right)^2, \tag{13}$$

where we take $a_0 = 1$ for the present time.

The bulk viscous pressure p_{visc} is described by Eckart's expression (Eckart, 1940) as follows

$$p_{visc} = \Pi = -\zeta U^{\mu}_{;\mu} = -3\zeta H, \tag{14}$$

where the non-negative quantity ζ is the bulk viscosity coefficient and $U^{\mu}_{;\mu}$ is the fluid expansion scalar which in the homogeneous and isotropic background reduces to $3H$, where $H = \dot{a}/a$ is the Hubble parameter and a is the scale factor of FRW metric.

Exact analytic cosmological solutions for a relativistic dissipative fluid can be found in the framework of the causal Israel-Stewart theory, which may be considered an alternate theory.

We assume the bulk viscosity given in the form (Brevik, 2002; Brevik and Gorbunova, 2005, 2008; Brevik *et al.*, 2010):

$$\zeta = \epsilon H^{-1}\rho_D, \tag{15}$$

where ϵ is a viscosity constant.

We consider that is an interaction between viscous MGDE and DM. The energy densities of MGDE and DM are not separately preserved in the presence of bulk viscosity and interaction, and thus the energy conservation equations for viscous MGDE and DM are given by the energy conservation equations

$$\dot{\rho_D} + 3H(1+\omega_D)\rho_D = 9\zeta H^2 - Q, \tag{16}$$

$$\dot{\rho_m} + 3H\rho_m = Q, \tag{17}$$

where $\omega_D = \frac{p_D}{\rho_D}$ is the EoS parameter of interacting viscous MGDE and Q represents the interaction term.

The interaction term Q can be, in general, an arbitrary function of cosmological parameters like the Hubble parameter and energy densities $Q(H\rho_m, H\rho_d)$ (Amendola, 2001; Banerjee and Pavón, 2007; Pavon and Zimdahl, 2005; Sheykhi, 2009; Zimdahl and Pavon, 2001). The positive b^2 is responsible for the energy transition from dark energy to dark matter. Sometimes this constant is taken in the range $[0,1]$ (Zhang and Zhu, 2006). Note that, if $b^2 = 0$, it represents the non-interacting FRW model while $b^2 = 1$ yields complete transfer of energy from dark energy to dark matter. Observations of cosmic microwave background and galactic clusters show that the coupling parameter $b^2 < 0.025$, *i.e.*, a small, but positive, constant of order unity (Ichiki *et al.*, 2008), a negative coupling parameter is avoided due to violation of thermodynamical laws. Therefore, the theoretical interacting models are phenomenologically consistent with the observations.

Thus, we take the explicit form of interaction term Q as

$$Q = 3b^2H(\rho_D + \rho_m). \tag{18}$$

Taking the time derivative of Eq. (6), and using the energy conservation Eqs. (16) and (17), we obtain

$$2H\dot{H} = \frac{8\pi\dot{G}}{3}(\rho_D+\rho_m) + \frac{2k}{a^2}H + \frac{8\pi G}{3}\left[9H^2\zeta - 3H(1+\omega_D)\rho_D - 3H\rho_m\right]. \tag{19}$$

Using the expressions of density parameters, Eqs. (7) to (9) and Eq. (15) and the relation $\dot{G} = HG'$, Eq. (19) can be expressed as

$$\frac{\dot{H}}{H^2} = (1+\Omega_k)\left(\frac{G'}{2G}\right) - \frac{3}{2}\left[(\omega_D - 3\epsilon)\Omega_D + (\Omega_D+\Omega_m) - \frac{2}{3}\Omega_k\right] \tag{20}$$

Above equation can be rewritten as

$$\frac{\dot{H}}{H^2} = (1+\Omega_k)\frac{\Delta G}{2} - \frac{3}{2}\left[1+\frac{\Omega_k}{3} + (\omega_D - 3\epsilon)\,\Omega_D\right], \tag{21}$$

where $\triangle G = \frac{G'}{G}$ arises from the varying nature of G and the prime denotes derivative with respect to $x \equiv \ln a$.

Now, taking the time derivative of Eq. (1), we get

$$\dot{\rho_D} = \dot{H}(\alpha+\beta H) + \beta H\dot{H}. \tag{22}$$

Using the Eqs. (1), (11) and (12), the above equation can be rewritten as

$$\dot{\rho_D} = \frac{\dot{H}}{H^2}\left(\frac{\Omega_D - \gamma + 1}{\Omega_D}\right)\rho_D H. \tag{23}$$

By substituting Eqs. (15), (18) and the above value of $\dot{\rho_D}$, Eq. (23) in the conservation Eq. (16), we obtain

$$\frac{\dot{H}}{H^2}\left(\frac{\Omega_D - \gamma + 1}{\Omega_D}\right)\rho_D H + 3(1+\omega_D)\rho_D H = 9\epsilon\rho_D H - 3b^2 H(\rho_D + \rho_m). \tag{24}$$

By substituting the value of $\frac{\dot{H}}{H^2}$, from Eq. (21) in above equation and after simplifying the same, we obtain the EoS parameter of the interacting viscous MGDE as

$$\begin{aligned}\omega_D &= \frac{1}{\Omega_D - \gamma - 1}\left[(1+\Omega_k)\left(\frac{\Omega_D - \gamma + 1}{\Omega_D}\right) \times \frac{\Delta G}{3}\right.\\ &+ \left. 1 - \frac{\Omega_k}{3} + \left(\frac{\gamma - 1}{\Omega_D}\right)\left(1 + \frac{\Omega_k}{3}\right) + 2b^2\left(\frac{1+\Omega_k}{\Omega_D}\right)\right] + 3\epsilon. \end{aligned} \tag{25}$$

We know that, the deceleration parameter plays vital role in cosmology, which describes cosmic dynamics for the late time acceleration. A positive value of the deceleration parameter characterizes a decelerating universe, while a negative value characterizes an accelerating universe.

The deceleration parameter q can be evaluated as

$$q = -1 - \frac{\dot{H}}{H^2}$$

$$= \frac{\gamma + 2(\Omega_D - 1) + (1+\Omega_k)\Delta G - \Omega_k + 3b^2(1+\Omega_k)}{(\Omega_D - \gamma - 1)}. \tag{26}$$

Taking derivative of Eq. (8) with respect to redshift $z = \left(\frac{1}{a} - 1\right)$, we obtain

$$\frac{d\Omega_D}{dz} = \frac{-1}{1+z}\left[\Omega_D\left(\frac{\dot{G}}{GH}\right) + \frac{8\pi G}{3H^2}\left(\dot{\rho_D} - 2\rho_D\frac{\dot{H}}{H}\right)\frac{1}{H}\right]. \tag{27}$$

Substituting the value of $\dot{\rho_D}$ from Eq. (23) in Eq. (27), and using the relation $\dot{G} = HG'$, above expression can be rewritten as

$$\frac{d\Omega_D}{dz} = \frac{-1}{1+z}\left[\Omega_D \frac{G'}{G} + (\Omega_D + \gamma - 1)\left(-\frac{\dot{H}}{H^2}\right)\right]. \tag{28}$$

Using the relation $\Delta G = G'/G$ with the value of q from Eq. (26) in the above expression, then we get

$$\frac{d\Omega_D}{dz} = \frac{-1}{1+z}\left[\Omega_D \triangle G + (\Omega_D + \gamma - 1)(q+1)\right]. \tag{29}$$

Substituting Eq. (21) in Eq. (28), we obtain the differential equation governing the MGDE density parameter as

$$\frac{d\Omega_D}{dz} = -\left(\frac{3}{1+z}\right)\left[\frac{\Omega_D \triangle G}{3} + \left(\frac{\Omega_D + \gamma - 1}{\Omega_D - \gamma - 1}\right)\right.$$
$$\left.\times\left(\Omega_D - 1 + (1+\Omega_k)\frac{\triangle G}{3} - \frac{\Omega_k}{3} + b^2(1+\Omega_k)\right)\right]. \tag{30}$$

From Eqs. (26) and (30), the deceleration and MGDE density parameters were found to be independent of the viscosity constant. The viscosity appeared only in the dynamical EoS parameter of MGDE which is given in Eq. (25).

It is very difficult to find the solution of differential Eq. (30) analytically, hence we solve this equation numerically. We obtain $\Omega_D(z)$ by solving the differential Eq. (30) numerically using the Eq. (13) and by taking $\Omega_{k0} = 0.01$, $\Omega_{D0} = 0.76$ for the present time (Cai *et al.*, 2012) with $\gamma = 1.105$ and $\Delta G = -0.0025$ (Jamil *et al.*, 2009). Variation of the MGDE density parameter $\Omega_D(z)$ against redshift $z = \left(\frac{1}{a} - 1\right)$ for different coupling constants b^2 is plotted in Figure 1, which shows that: (i) for a given b^2, Ω_D increases during history of the universe. (ii) At early and late times, Ω_D increases and decreases, respectively, with increasing b^2.

Similarly, we have plotted, the evolutionary behavior of the deceleration parameter q given in Eq. (26) for different coupling constants b^2 respectively in Figure 2, which shows that: (i) At early times, the universe transitions from a matter dominated epoch to the de-Sitter phase, *i.e.* $q = -1$, as expected in the future. The outcome for $b^2 = 0$ is in agreement with that obtained by Cai *et al.* (2012). (ii) We have a cosmic deceleration $q > 0$ to acceleration $q < 0$ transition for $b^2 = 0.0, 0.02$ and 0.04 at $z = 0.75, 0.91$ and 1.10, respectively, which is compatible with the observations (Ishida *et al.*, 2008). (iii) For a given z, whereas q decreases as b^2 increases, q becomes independent of b^2 in the far future.

The variation of the EoS parameter of interacting viscous MGDE given in Eq. (25) for different coupling b^2 and viscosity constants ϵ are expressed in Figures 3, 4 and 5 respectively. Figures 3 and 4 indicate that: (i) in the absence of

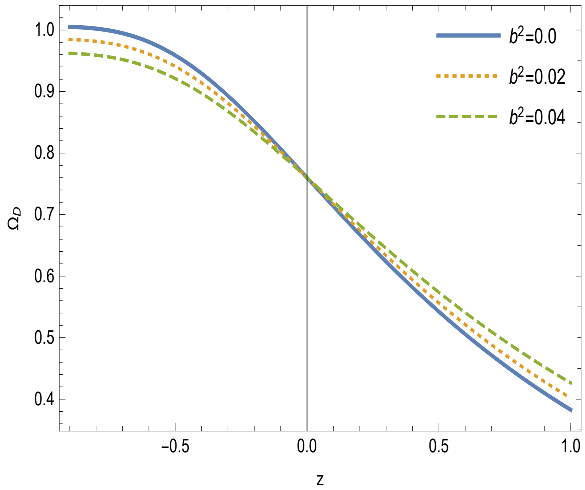

Fig. 1 – MGDE density parameter Ω_D versus redshift z for different coupling constants b^2, with auxiliary parameters $\Omega_{k0} = 0.01$, $\Omega_{D0} = 0.76$, $\gamma = 1.105$ and $\Delta G = -0.0025$ based on Eq. (30).

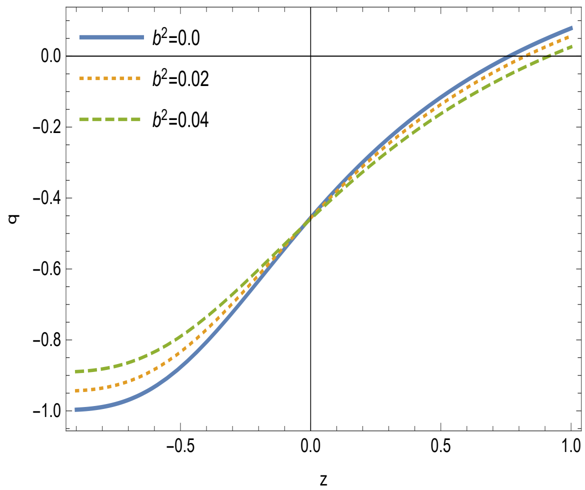

Fig. 2 – Deceleration parameter q versus redshift z for different coupling constants b^2, with auxiliary parameters $\Omega_{k0} = 0.01$, $\Omega_{D0} = 0.76$, $\gamma = 1.105$ and $\Delta G = -0.0025$ based on Eq. (26).

viscosity ($\epsilon = 0$), $\omega_D(z)$ varies from the quintessence phases ($\omega_D > -1$) to the phantom regime ($\omega_D < -1$); (ii) when interaction is absent ($b^2 = 0$), $\omega_D(z)$ varies from $\omega_D > -1$ to $\omega_D = -1$, which is similar to the freezing quintessence model (Caldwell and Linder, 2005); (iii) ω_D decreases and increases with increases of b^2 and ϵ, respectively, for a given z. Note that the result for $b^2 = \epsilon = 0$ is in accordance with that obtained by Cai *et al.* (2012). Figure 5 shows that both b^2 and ϵ are not zero and there are opposite effects on ω_D for interaction and viscosity.

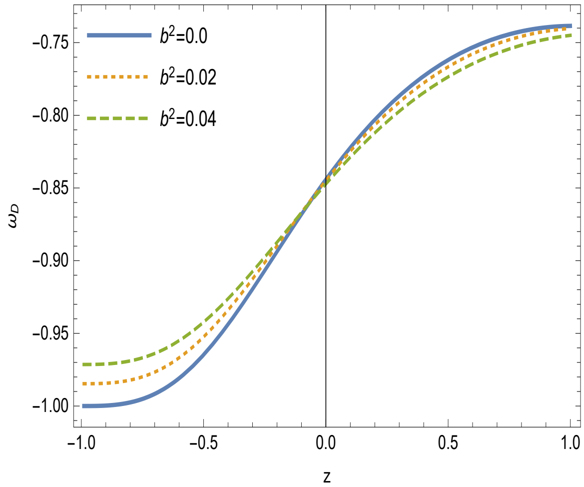

Fig. 3 – EoS parameter ω_D versus redshift z for different coupling constants b^2 and viscosity constant $\epsilon = 0$, with auxiliary parameters $\Omega_{k0} = 0.01$, $\Omega_{D0} = 0.76$, $\gamma = 1.105$ and $\Delta G = -0.0025$ based on Eq. (25).

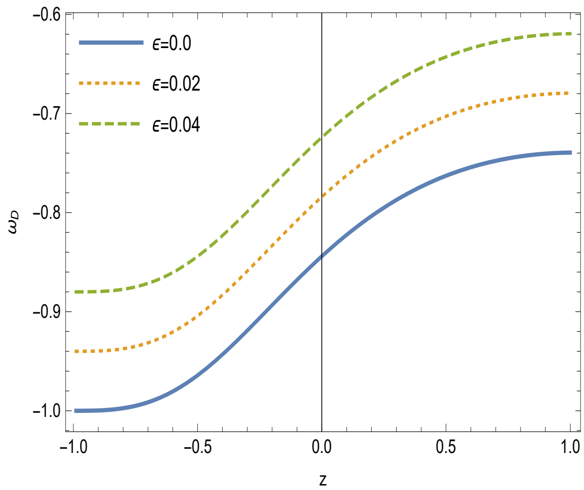

Fig. 4 – EoS parameter ω_D versus redshift z for different viscosity constants ϵ and coupling constant $b^2 = 0$, with auxiliary parameters $\Omega_{k0} = 0.01$, $\Omega_{D0} = 0.76$, $\gamma = 1.105$ and $\Delta G = -0.0025$ based on Eq. (25).

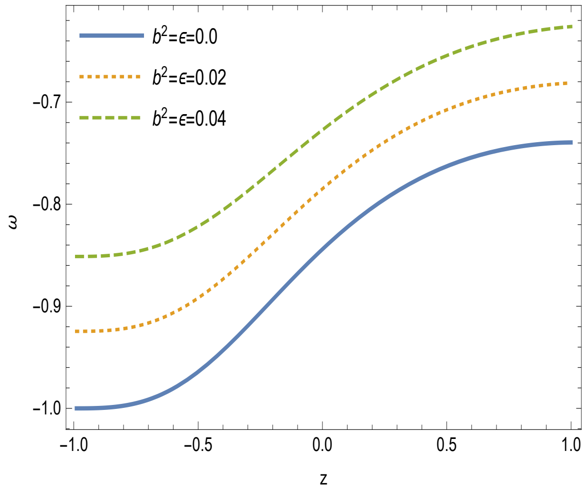

Fig. 5 – EoS parameter ω_D versus redshift z for different coupling constants b^2 and viscosity constants ϵ, with auxiliary parameters $\Omega_{k0} = 0.01$, $\Omega_{D0} = 0.76$, $\gamma = 1.105$ and $\Delta G = -0.0025$ based on Eq. (25).

3. CORRESPONDENCE BETWEEN VISCOUS MGDE AND SCALAR FIELD MODELS OF DE

Scalar fields are important areas in particle physics including string theory and these act as a candidate for dark energy. Therefore, we study a correspondence between the interacting viscous MGDE model and quintessence, tachyon, k-essence and dilaton scalar field models. In this section, we establish a correspondence between the viscous MGDE and various scalar field models by comparing their respective energy densities and EoS parameters and then reconstruct the dynamics and potential of scalar field models of DE.

3.1. QUINTESSENCE SCALAR FIELD MODEL

Quintessence is described by an ordinary scalar field ϕ minimally coupled to gravity. The quintessence scalar field model of dark energy was proposed to justify the late-time acceleration of the universe.
The action for quintessence is given by Caldwell *et al.* (1998); Copeland and Sheykhi (2006); Ratra and Peebles (1988).

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial\mu\phi\partial\nu\phi - V(\phi)\right]. \tag{31}$$

The energy density and pressure of the quintessence scalar field are given by

Copeland and Sheykhi (2006)

$$p_Q = \frac{1}{2}\dot{\phi}^2 - V(\phi), \tag{32}$$

$$\rho_Q = \frac{1}{2}\dot{\phi}^2 + V(\phi). \tag{33}$$

Hence, using Eqs. (32) and (33), EoS parameter for quintessence scalar field is obtained as

$$\omega_Q = \frac{p_Q}{\rho_Q} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}. \tag{34}$$

Similarly, using Eqs. (32) and (33) we obtained the expressions for kinetic energy and quintessence potential energy as

$$\dot{\phi}^2 = (1+\omega_Q)\,\rho_Q, \tag{35}$$

$$V(\phi) = \frac{1}{2}(1-\omega_Q)\,\rho_Q. \tag{36}$$

To establish a correspondence between MGDE and scalar field models, we take $\rho_Q = \rho_D$ and $\omega_Q = \omega_D$.

Hence, equating Eq. (34) with EoS parameter (25) and also equating Eq. (33) and Eq. (1), we have

$$\dot{\phi}^2 = (1+\omega_D)\,\rho_D. \tag{37}$$

$$V(\phi) = \frac{1}{2}(1-\omega_D)\,\rho_D. \tag{38}$$

By using above Eq. (38), we obtain

$$\begin{aligned} V(\phi) \quad = \quad & \frac{\rho_D}{2}\Bigg[\frac{1}{\Omega_D-\gamma-1}\Bigg\{\Omega_D-\gamma-1-(1+\Omega_k)\left(\frac{\Omega_D-\gamma+1}{\Omega_D}\right)\frac{\Delta G}{3} \\ - \quad & 1+\frac{\Omega_k}{3}-\left(\frac{\gamma-1}{\Omega_D}\right)\left(1+\frac{\Omega_k}{3}\right)-2b^2\left(\frac{1+\Omega_k}{\Omega_D}\right)\Bigg\}-3\epsilon\Bigg]. \end{aligned} \tag{39}$$

Hence, scalar potential $V(\phi)$ can be expressed as

$$\begin{aligned} V(\phi) = \frac{8\pi G\alpha^2\Omega_D}{6\,(\Omega_D+\gamma-1)^2}&\Bigg[\frac{1}{\Omega_D-\gamma-1}\Bigg\{\Omega_D-\gamma-1-(1+\Omega_k)\left(\frac{\Omega_D-\gamma+1}{\Omega_D}\right)\frac{\Delta G}{3} \\ & -1+\frac{\Omega_k}{3}-\left(\frac{\gamma-1}{\Omega_D}\right)\left(1+\frac{\Omega_k}{3}\right)-2b^2\left(\frac{1+\Omega_k}{\Omega_D}\right)\Bigg\}-3\epsilon\Bigg]. \end{aligned} \tag{40}$$

Similarly, using Eqs. (37), we obtain the kinetic energy as

$$\begin{aligned} \dot{\phi}^2 = \rho_D&\Bigg[\frac{1}{\Omega_D-\gamma-1}\Bigg\{\Omega_D-\gamma-1+(1+\Omega_k)\left(\frac{\Omega_D-\gamma+1}{\Omega_D}\right)\frac{\Delta G}{3} \\ & +1-\frac{\Omega_k}{3}+\left(\frac{\gamma-1}{\Omega_D}\right)\left(1+\frac{\Omega_k}{3}\right)+2b^2\left(\frac{1+\Omega_k}{\Omega_D}\right)\Bigg\}+3\epsilon\Bigg]. \end{aligned} \tag{41}$$

Hence, kinetic energy ($\dot{\phi}^2$), can also be expressed as

$$\dot{\phi}^2 = \frac{8\pi G\alpha^2\Omega_D}{3\left(\Omega_D+\gamma-1\right)^2}\left[\frac{1}{\Omega_D-\gamma-1}\left\{\Omega_D-\gamma-1+(1+\Omega_k)\left(\frac{\Omega_D-\gamma+1}{\Omega_D}\right)\frac{\Delta G}{3}\right.\right.$$
$$\left.\left.+1-\frac{\Omega_k}{3}+\left(\frac{\gamma-1}{\Omega_D}\right)\left(1+\frac{\Omega_k}{3}\right)+2b^2\left(\frac{1+\Omega_k}{\Omega_D}\right)\right\}+3\epsilon\right]. \quad (42)$$

Integrating Eq. (42) with respect to scale factor a, the evolution of quintessence scalar field corresponding to MGDE is given by

$$\phi(a)-\phi(a_0) = \sqrt{\frac{3}{8\pi G}}\int_{a_0}^{a}\sqrt{\Omega_D}\left[\frac{1}{\Omega_D-\gamma-1}\left\{\Omega_D-\gamma-1+(1+\Omega_k)\right.\right.$$
$$\times\left(\frac{\Omega_D-\gamma+1}{\Omega_D}\right)\frac{\Delta G}{3}+1-\frac{\Omega_k}{3}+\left(\frac{\gamma-1}{\Omega_D}\right)\left(1+\frac{\Omega_k}{3}\right)+2b^2\left(\frac{1+\Omega_k}{\Omega_D}\right)\Bigg\}$$
$$\left.+3\epsilon\right]^{\frac{1}{2}}\frac{da}{a}, \quad (43)$$

where a_0 is the scale factor at the present time. Using the Eq. (43) and numerical solution of the differential Eq. (30), we obtained the evolutionary bahavior of the quintessence MGDE scalar field. We have plotted the variation of quintessence scalar field, Eq. (40), versus redshift in Figures 6, 7 and 8 respectively. Figures 6 and 7, show that: (i) for a given b^2 or ϵ, scalar field ϕ increases during history of the universe. (ii) For a given z, ϕ decreases and increases with increasing b^2 and ϵ, respectively. Figure 6 shows that for $b^2 = 0.0,\ 0.02$ and 0.04 at $z < -0.97, -0.57$ and -0.41, respectively, ϕ becomes pure imaginary, *i.e.* $\dot{\phi}^2 < 0$, and does not show itself in Figure 6. For $\dot{\phi}^2 < 0$ the MGDE scalar field behaves as a phantom-type scalar field (Caldwell, 2002). Figure 8 clarifies that interaction and viscosity have opposite effects on ϕ. For $b^2 = \epsilon$, the effect of viscosity is more than the interaction, at late times they neutralize the effect of each other.

In Figures 9 to 11, the variation of quintessence MGDE potential, Eq. (40), versus redshift is plotted for different b^2 and ϵ. Figures 9 and 10, show that: (i) for a given b^2 or ϵ, the quintessence MGDE potential $V(\phi)$ decreases during history of the universe. (ii) For a given z, $V(\phi)$ increases and decreases with increasing b^2 and ϵ, respectively. Figure 11 shows that in the presence of both interaction and viscosity, although for $b^2 = \epsilon$ at early times the effect of viscosity is more than the interaction, at late times they neutralize the effect of each other.

3.2. TACHYON SCALAR FIELD MODEL

The tachyon scalar field model in class of string theories can act as a source of dark energy depending on the form of the tachyon potential. The effective Lagrangian for the tachyon scalar field is given by Padmanabhan (2002); Padmanabhan

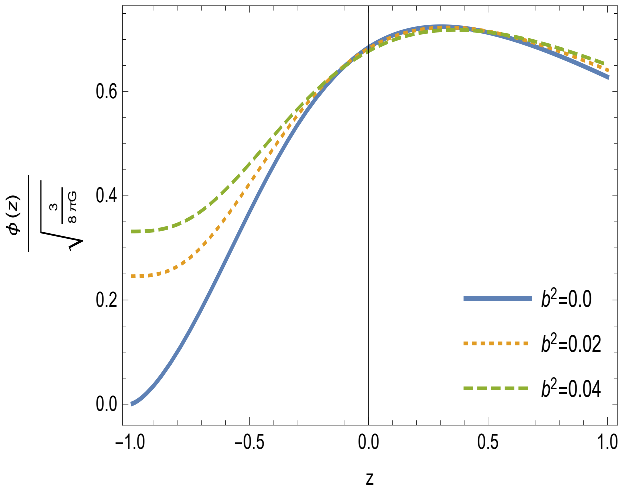

Fig. 6 – Quintessence MGDE scalar field versus redshift for different coupling constants b^2 and viscosity constants $\epsilon = 0$ with auxiliary parameters $\Omega_{k0} = 0.01$, $\Omega_{D0} = 0.76$, $\gamma = 1.105$ and $\Delta G = -0.0025$ based on Eq. (43).

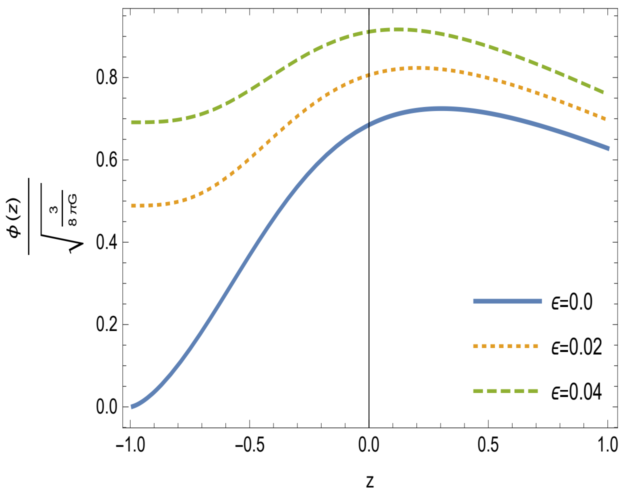

Fig. 7 – Quintessence MGDE scalar field versus redshift for different viscosity constants ϵ and coupling constants $b^2 = 0$ with auxiliary parameters $\Omega_{k0} = 0.01$, $\Omega_{D0} = 0.76$, $\gamma = 1.105$ and $\Delta G = -0.0025$ based on Eq. (43).

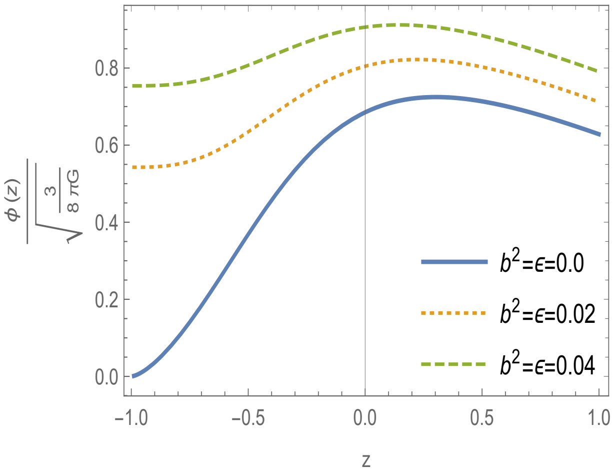

Fig. 8 – Quintessence MGDE scalar field versus redshift for different coupling constants b^2 and viscosity constants ϵ with auxiliary parameters $\Omega_{k0} = 0.01$, $\Omega_{D0} = 0.76$, $\gamma = 1.105$ and $\Delta G = -0.0025$ based on Eq. (43).

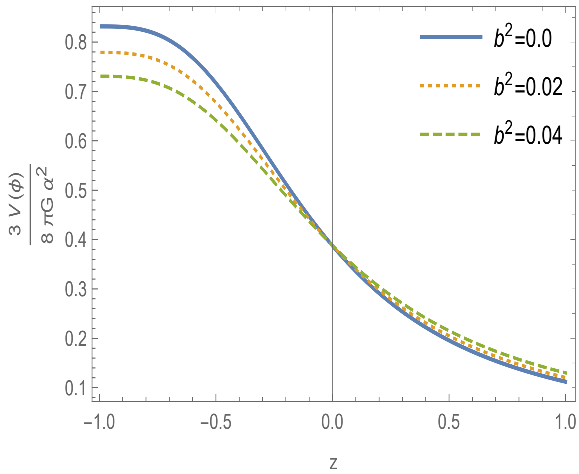

Fig. 9 – Quintessence MGDE potential versus redshift for different coupling constants b^2 and viscosity constants $\epsilon = 0$ with auxiliary parameters $\Omega_{k0} = 0.01$, $\Omega_{D0} = 0.76$, $\gamma = 1.105$ and $\Delta G = -0.0025$ based on Eq. (40).

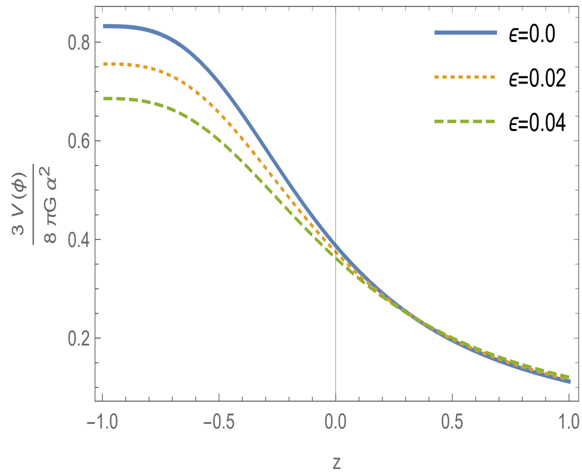

Fig. 10 – Quintessence MGDE potential versus redshift for different viscosity constants ϵ and coupling constants $b^2 = 0$ with auxiliary parameters $\Omega_{k0} = 0.01$, $\Omega_{D0} = 0.76$, $\gamma = 1.105$ and $\Delta G = -0.0025$ based on Eq. (40).

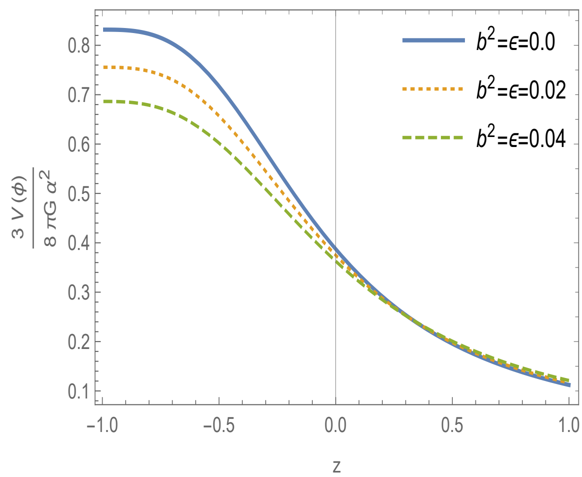

Fig. 11 – Quintessence MGDE potential versus redshift for different coupling constants b^2 and viscosity constants ϵ with auxiliary parameters $\Omega_{k0} = 0.01$, $\Omega_{D0} = 0.76$, $\gamma = 1.105$ and $\Delta G = -0.0025$ based on Eq. (40).

and Choudhury (2002); Sen (1999, 2002a,b)

$$\mathcal{L} = -V(\phi)\sqrt{1+\partial_\mu\phi\partial^\mu\phi}, \tag{44}$$

where ϕ is the tachyon scalar field and $V(\phi)$ is the tachyon potential.

The energy density and pressure of tachyon scalar field are given by

$$\rho_T = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}}, \tag{45}$$

$$p_T = -V(\phi)\sqrt{1-\dot{\phi}^2}. \tag{46}$$

Therefore, the EoS parameter of tachyon scalar field is obtained as

$$\omega_T = \frac{p_T}{\rho_T} = \dot{\phi}^2 - 1. \tag{47}$$

As we establish correspondence between interacting viscous MGDE and tachyon DE, equating Eq. (47) with (25) *i.e.* $\omega_T = \omega_D$ and also, equating Eq. (45) with (1), $\rho_T = \rho_D$, we obtain

$$\omega_D = \dot{\phi}^2 - 1, \tag{48}$$

$$V(\phi) = \rho_D\sqrt{1-\dot{\phi}^2}. \tag{49}$$

Substituting ω_D from the Eq. (25) in Eq. (48), we obtain the expression for kinetic energy as

$$\dot{\phi}^2 = \frac{1}{\Omega_D-\gamma-1}\left[\Omega_D-\gamma-\frac{\Omega_\kappa}{3}+(1+\Omega_\kappa)\left(\frac{\Omega_D-\gamma+1}{\Omega_D}\right)\frac{\Delta G}{3}\right.$$
$$\left.+\frac{\gamma-1}{\Omega_D}\left(1+\frac{\Omega_\kappa}{3}\right)+2b^2\left(\frac{1+\Omega_\kappa}{\Omega_D}\right)\right]+3\epsilon. \tag{50}$$

Integrating Eq. (50) with respect to scale factor a, we obtain the evolutionary form of the modified ghost tachyon scalar field as

$$\phi(a)-\phi(a^0) = \frac{3}{8\pi G\alpha}\int_{a_0}^{a}(\Omega_D+\gamma-1)\left[\frac{1}{\Omega_D-\gamma-1}\left\{\Omega_D-\gamma-\frac{\Omega_\kappa}{3}+(1+\Omega_\kappa)\right.\right.$$
$$\left.\left.\times\left(\frac{\Omega_D-\gamma+1}{\Omega_D}\right)\frac{\Delta G}{3}+\frac{\gamma-1}{\Omega_D}\left(1+\frac{\Omega_\kappa}{3}\right)+2b^2\left(\frac{1+\Omega_\kappa}{\Omega_D}\right)\right\}+3\epsilon\right]^{\frac{1}{2}}\frac{da}{a}. \tag{51}$$

Furthermore, substituting the value of ρ_D from Eq. (1) and $\dot{\phi}^2$ from Eq. (50) in Eq. (49), we determine the tachyon potential

$$V(\phi) = \rho_D\left[\frac{1}{\Omega_D-\gamma-1}\left\{1-\frac{\Omega_\kappa}{3}+(1+\Omega_\kappa)\left(\frac{\Omega_D-\gamma+1}{\Omega_D}\right)\frac{\Delta G}{3}\right.\right.$$
$$\left.\left.+\frac{\gamma-1}{\Omega_D}\left(1+\frac{\Omega_\kappa}{3}\right)+2b^2\left(\frac{1+\Omega_\kappa}{\Omega_D}\right)\right\}+3\epsilon\right]^{\frac{1}{2}}. \tag{52}$$

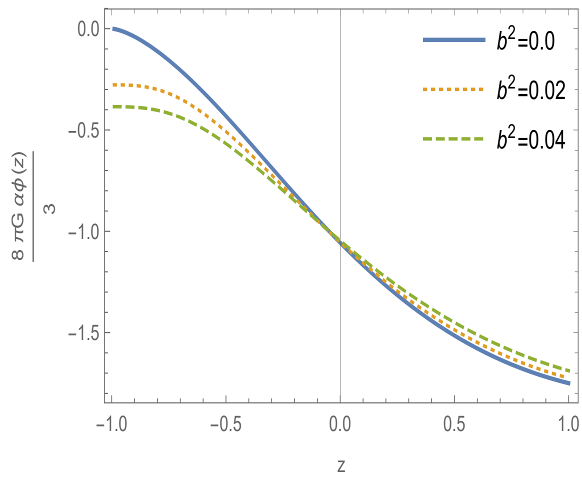

Fig. 12 – Tachyon MGDE scalar field versus redshift for different coupling constants b^2 and viscosity constant $\epsilon = 0$, with auxiliary parameters $\Omega_{k0} = 0.01$, $\Omega_{D0} = 0.76$, $\gamma = 1.105$ and $\Delta G = -0.0025$ based on Eq. (51).

Hence, the tachyon potential $V(\phi)$, can also be expressed as

$$V(\phi) = \frac{8\pi G\alpha^2\Omega_D}{3(\Omega_D+\gamma-1)^2}\left[\frac{1}{\Omega_D-\gamma-1}\left\{1-\frac{\Omega_\kappa}{3}+(1+\Omega_\kappa)\left(\frac{\Omega_D-\gamma+1}{\Omega_D}\right)\frac{\Delta G}{3}\right.\right.$$
$$\left.\left.+\frac{\gamma-1}{\Omega_D}\left(1+\frac{\Omega_\kappa}{3}\right)+2b^2\left(\frac{1+\Omega_\kappa}{\Omega_D}\right)\right\}+3\epsilon\right]^{\frac{1}{2}}. \quad (53)$$

In Figures 12 to 14, we have plotted the variation of tachyon scalar field, Eq. (51), versus redshift for different coupling constants b^2 and viscosity constants ϵ. Also, the evolution of tachyon MGDE potential, Eq. (53), is plotted in Figures 15 to 17 respectively. Figures 12 to 17, show that the scalar field and potential of the tachyon MGDE scalar field model behave like the quintessence model of DE.

3.3. K-ESSENCE SCALAR FIELD MODEL

The $k-$essence scalar field model of dark energy is characterized by a scalar field with a non-canonical kinetic energy. The scalar field action for k-essence is a function of ϕ and $\chi = \dot{\phi}^2/2$ and is given as (Armendariz *et al.*, 1999, 2000, 2001; Chiba *et al.*, 2000; Garriga and Mukhanov, 1999):

$$\mathcal{S} = d^4x\sqrt{-g}\,p(\phi,\chi), \quad (54)$$

where the Lagrangian density $p(\phi,\chi)$ corresponds to a pressure with non-canonical kinetic term as

$$p_K(\phi,\chi) = f(\phi)\left(-\chi+\chi^2\right). \quad (55)$$

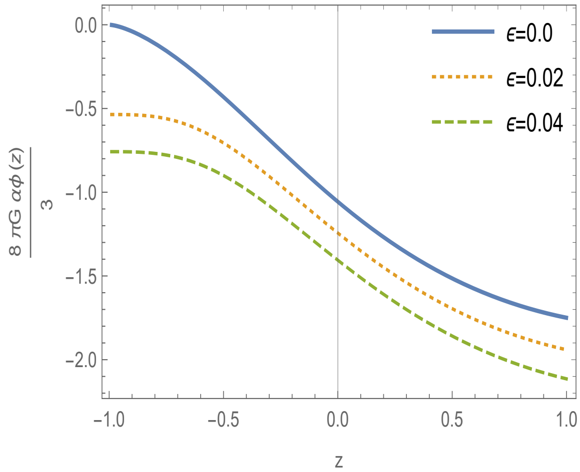

Fig. 13 – Tachyon MGDE scalar field versus redshift for different viscosity constants ϵ and coupling constant $b^2 = 0$, with auxiliary parameters $\Omega_{k0} = 0.01$, $\Omega_{D0} = 0.76$, $\gamma = 1.105$ and $\Delta G = -0.0025$ based on Eq. (53).

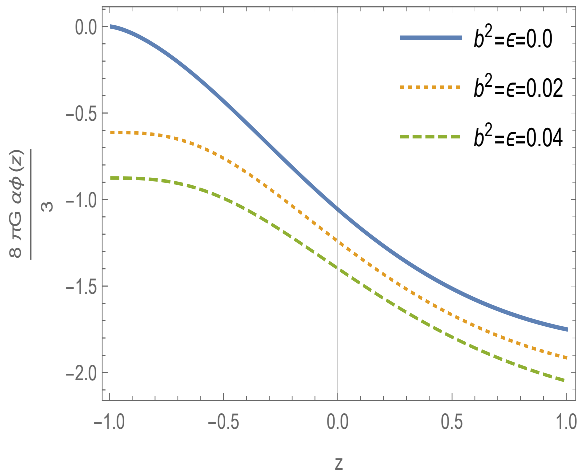

Fig. 14 – Tachyon MGDE scalar field versus redshift for different coupling constants b^2 and viscosity constants ϵ, with auxiliary parameters $\Omega_{k0} = 0.01$, $\Omega_{D0} = 0.76$, $\gamma = 1.105$ and $\Delta G = -0.0025$ based on Eq. (51).

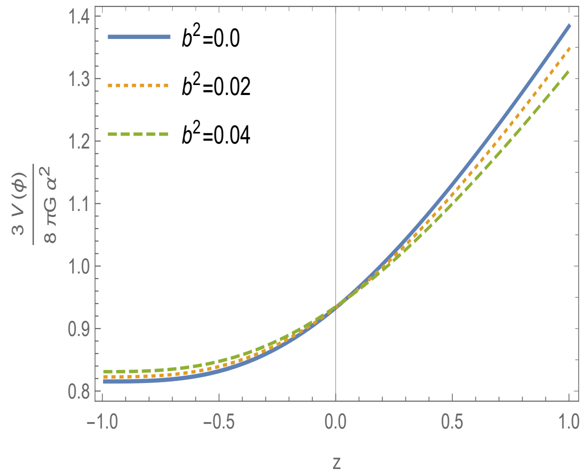

Fig. 15 – Tachyon MGDE potential versus redshift for different coupling constants b^2 and viscosity constant $\epsilon = 0$, with auxiliary parameters $\Omega_{k0} = 0.01$, $\Omega_{D0} = 0.76$, $\gamma = 1.105$ and $\Delta G = -0.0025$ based on Eq. (52).

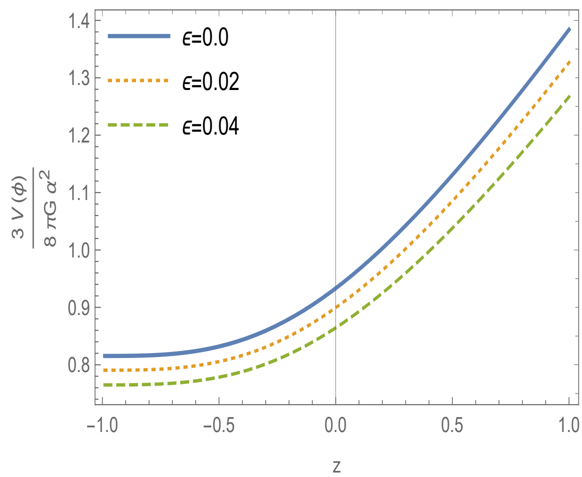

Fig. 16 – Tachyon MGDE potential versus redshift for different viscosity constants $\epsilon = 0$ and coupling constants b^2, with auxiliary parameters $\Omega_{k0} = 0.01$, $\Omega_{D0} = 0.76$, $\gamma = 1.105$ and $\Delta G = -0.0025$ based on Eq. (52).

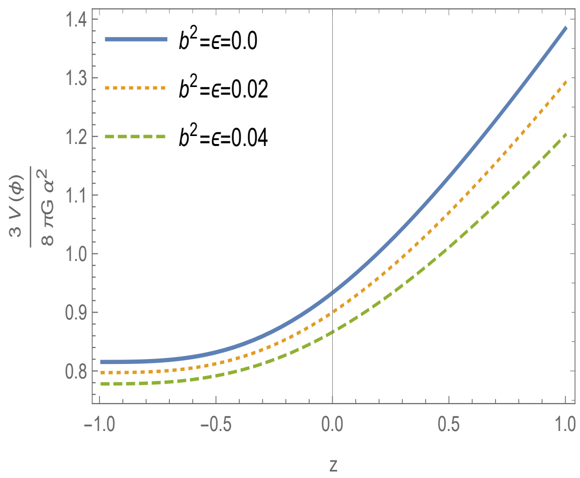

Fig. 17 – Tachyon MGDE potential versus redshift for different coupling constants b^2 and viscosity constants ϵ, with auxiliary parameters $\Omega_{k0} = 0.01$, $\Omega_{D0} = 0.76$, $\gamma = 1.105$ and $\Delta G = -0.0025$ based on Eq. (52).

Also, the energy density of the k-essence field is given by

$$\rho_K(\phi,\chi) = f(\phi)\left(-\chi + 3\chi^2\right). \tag{56}$$

The EoS parameter of the k-essence scalar field is obtained as

$$\omega_K = \frac{\chi - 1}{3\chi - 1}. \tag{57}$$

Equating Eq. (57) with EoS parameter given in Eq. (25), $\omega_K = \omega_D$, we obtain the solution for χ as

$$\begin{aligned}\chi = \Bigg[&2+\gamma+(1+\Omega_\kappa)\left(\frac{\Omega_D-\gamma+1}{\Omega_D}\right)\frac{\Delta G}{3} - \frac{\Omega_\kappa}{3} + \frac{\gamma-1}{\Omega_D}\left(1+\frac{\Omega_\kappa}{3}\right)\\ &+2b^2\left(\frac{1+\Omega_\kappa}{\Omega_D}\right) - \Omega_D + 3\epsilon\left(\Omega_D-\gamma-1\right)\Bigg]\\ \times\Bigg[&4+\gamma+\left(\frac{\Omega_D-\gamma+1}{\Omega_D}\right)\frac{\Delta G}{3} - \Omega_\kappa + \frac{\gamma-1}{\Omega_D}(3+\Omega_\kappa) + 6b^2\left(\frac{1+\Omega_\kappa}{\Omega_D}\right)\\ &-\Omega_D + 9\epsilon\left(\Omega_D-\gamma-1\right)\Bigg]^{-1}.\end{aligned} \tag{58}$$

Using $\dot{\phi}^2 = 2\chi$ and Eq. (58), the evolutionary form of the k-essence scalar field is obtained as

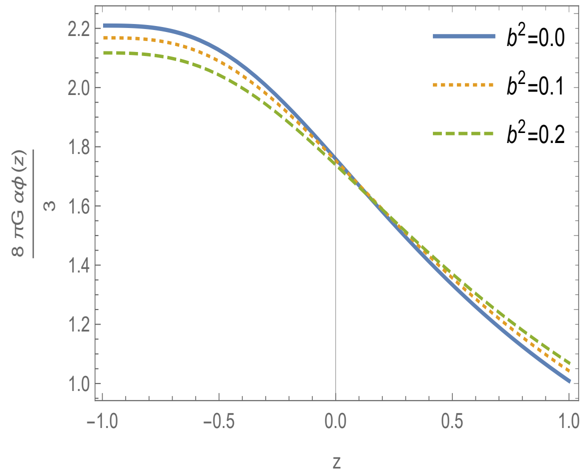

Fig. 18 – k-essence MGDE scalar field versus redshift for different coupling constants b^2 and viscosity constant $\epsilon = 0$, with auxiliary parameters $\Omega_{k0} = 0.01$, $\Omega_{D0} = 0.76$, $\gamma = 1.105$ and $\Delta G = -0.0025$ based on Eq. (59).

$$\phi(a) - \phi(a_0) = \frac{3}{8\pi G\alpha}\int_{a_0}^{a}(\Omega_D - \gamma - 1)\times\left[4 + 2\gamma + 2(1+\Omega_\kappa)\left(\frac{\Omega_D - \gamma + 1}{\Omega_D}\right)\frac{\Delta G}{3}\right.$$
$$\left. - \frac{2\Omega_\kappa}{3} + 2\left(\frac{\gamma - 1}{\Omega_D}\right)\times\left(1 + \frac{\Omega_\kappa}{3}\right) + 4b^2\left(\frac{1+\Omega_\kappa}{\Omega_D}\right) - 2\Omega_D + 6\epsilon(\Omega_D - \gamma - 1)\right]^{\frac{1}{2}}$$
$$\left[4 + \gamma + \left(\frac{\Omega_D - \gamma + 1}{\Omega_D}\right)\frac{\Delta G}{3} - \Omega_\kappa + \frac{\gamma - 1}{\Omega_D}(3 + \Omega_\kappa) + 6b^2\left(\frac{1+\Omega_\kappa}{\Omega_D}\right) - \Omega_D\right.$$
$$\left. + 9\epsilon(\Omega_D - \gamma - 1)\right]^{-\frac{1}{2}}\frac{da}{a}. \quad (59)$$

The evolutionary behavior of the k-essence MGDE scalar field for different coupling constants b^2 and viscosity constant $\epsilon = 0$ is plotted in Figures 18 to 20. These figures, show that the k-essence MGDE scalar field increases during history of the universe.

3.4. DILATON SCALAR FIELD MODEL

The dilaton scalar field is an interesting attempt to explain the origin of dark energy using string theory. For the dilaton dark energy model, the pressure and energy densities are given by (Gasperini *et al.*, 2002; Arkani *et al.*, 2004; Elizalde *et*

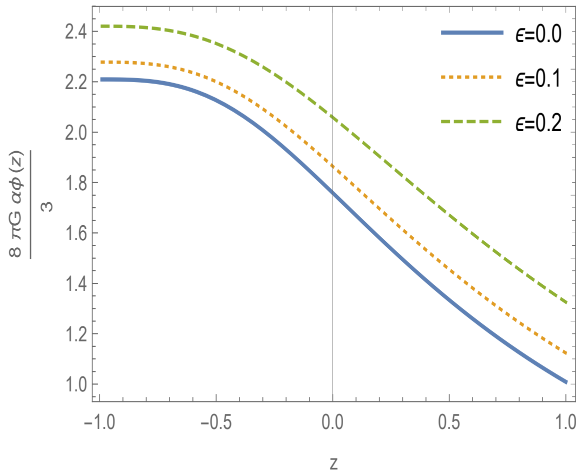

Fig. 19 – k-essence MGDE scalar field versus redshift for different viscosity constants ϵ and coupling constant $b^2 = 0$, with auxiliary parameters $\Omega_{k0} = 0.01$, $\Omega_{D0} = 0.76$, $\gamma = 1.105$ and $\Delta G = -0.0025$ based on Eq. (59).

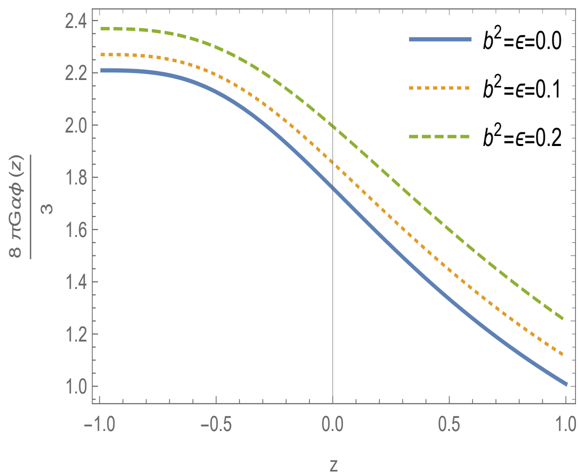

Fig. 20 – K-essence MGDE scalar field versus redshift for different coupling constants b^2 and viscosity constants ϵ, with auxiliary parameters $\Omega_{k0} = 0.01$, $\Omega_{D0} = 0.76$, $\gamma = 1.105$ and $\Delta G = -0.0025$ based on Eq. (59).

al., 2008):

$$p_{\mathcal{D}} = -\chi + ce^{\lambda\phi}\chi^2, \tag{60}$$

$$\rho_{\mathcal{D}} = -\chi + 3ce^{\lambda\phi}\chi^2, \tag{61}$$

where c and λ are positive constants and $\chi = \dot{\phi}^2/2$ is the dilaton kinetic energy.

The EoS parameter of the dilaton dark energy can be written as

$$\omega_{\mathcal{D}} = \frac{1 - ce^{\lambda\phi}\chi}{1 - 3ce^{\lambda\phi}\chi}. \tag{62}$$

In Eq. (62) with EoS, equating parameter (25), $\omega_{\mathcal{D}} = \omega_D$, we obtain

$$\begin{aligned} ce^{\lambda\phi}\chi = &\left[2 + \gamma + (1+\Omega_\kappa)\left(\frac{\Omega_D - \gamma + 1}{\Omega_D}\right)\frac{\Delta G}{3} - \frac{\Omega_\kappa}{3} + \frac{\gamma - 1}{\Omega_D}\left(1 + \frac{\Omega_\kappa}{3}\right)\right. \\ &\left. + 2b^2\left(\frac{1+\Omega_\kappa}{\Omega_D}\right) - \Omega_D + 3\epsilon\left(\Omega_D - \gamma - 1\right)\right] \\ &\left[4 + \gamma + \left(\frac{\Omega_D - \gamma + 1}{\Omega_D}\right)\frac{\Delta G}{3} - \Omega_\kappa + \frac{\gamma - 1}{\Omega_D}(3 + \Omega_\kappa)\right. \\ &\left. + 6b^2\left(\frac{1+\Omega_\kappa}{\Omega_D}\right) - \Omega_D + 9\epsilon\left(\Omega_D - \gamma - 1\right)\right]^{-1}. \end{aligned} \tag{63}$$

Using $\dot{\phi}^2 = 2\chi$, we can express the above equations as

$$\begin{aligned} e^{\frac{\lambda\phi}{2}}\dot{\phi} = \frac{1}{\sqrt{c}} &\left[4 + 2\gamma + 2(1+\Omega_\kappa)\left(\frac{\Omega_D - \gamma + 1}{\Omega_D}\right)\frac{\Delta G}{3} - \frac{2\Omega_\kappa}{3} + 2\left(\frac{\gamma - 1}{\Omega_D}\right)\right. \\ &\left. \times\left(1 + \frac{\Omega_\kappa}{3}\right) + 4b^2\left(\frac{1+\Omega_\kappa}{\Omega_D}\right) - 2\Omega_D + 6\epsilon\left(\Omega_D - \gamma - 1\right)\right]^{\frac{1}{2}} \\ &\left[4 + \gamma + \left(\frac{\Omega_D - \gamma + 1}{\Omega_D}\right)\frac{\Delta G}{3} - \Omega_\kappa + \frac{\gamma - 1}{\Omega_D}(3 + \Omega_\kappa) + 6b^2\left(\frac{1+\Omega_\kappa}{\Omega_D}\right)\right. \\ &\left. - \Omega_D + 9\epsilon\left(\Omega_D - \gamma - 1\right)\right]^{-\frac{1}{2}}. \end{aligned} \tag{64}$$

Integrating Eq. (64) with respect to scale factor a, we obtain

$$e^{\frac{\lambda\phi(a)}{2}} = e^{\frac{\lambda\phi(a_0)}{2}} + \frac{\lambda}{2\sqrt{c}}\int_{a_0}^{a}\frac{1}{H}\frac{da}{a}\left[4+2\gamma+2\left(1+\Omega_\kappa\right)\left(\frac{\Omega_D-\gamma+1}{\Omega_D}\right)\frac{\Delta G}{3}\right.$$
$$\left.-\frac{2\Omega_\kappa}{3}+2\left(\frac{\gamma-1}{\Omega_D}\right)\times\left(1+\frac{\Omega_\kappa}{3}\right)+4b^2\left(\frac{1+\Omega_\kappa}{\Omega_D}\right)-2\Omega_D+6\epsilon\left(\Omega_D-\gamma-1\right)\right]^{\frac{1}{2}}$$
$$\left[4+\gamma+\left(\frac{\Omega_D-\gamma+1}{\Omega_D}\right)\frac{\Delta G}{3}-\Omega_\kappa+\frac{\gamma-1}{\Omega_D}\left(3+\Omega_\kappa\right)+6b^2\left(\frac{1+\Omega_\kappa}{\Omega_D}\right)-\Omega_D\right.$$
$$\left.+9\epsilon\left(\Omega_D-\gamma-1\right)\right]^{-\frac{1}{2}}. \quad (65)$$

Finally, the evolutionary form of the dilaton scalar field is obtained as

$$\phi(a) = \frac{2}{\lambda}\ln\left\{e^{\frac{\lambda\phi(a_0)}{2}} + \frac{3\lambda}{8\pi G\alpha\sqrt{c}}\int_{a_0}^{a}\left(\Omega_D+\gamma-1\right)\times\left[4+2\gamma+2\left(1+\Omega_\kappa\right)\right.\right.$$
$$\times\left(\frac{\Omega_D-\gamma+1}{\Omega_D}\right)\frac{\Delta G}{3}-\frac{2\Omega_\kappa}{3}+2\left(\frac{\gamma-1}{\Omega_D}\right)\left(1+\frac{\Omega_\kappa}{3}\right)+4b^2\left(\frac{1+\Omega_\kappa}{\Omega_D}\right)-2\Omega_D$$
$$\left.+6\epsilon\left(\Omega_D-\gamma-1\right)\right]^{\frac{1}{2}}\left[4+\gamma+\left(\frac{\Omega_D-\gamma+1}{\Omega_D}\right)\frac{\Delta G}{3}-\Omega_\kappa+\frac{\gamma-1}{\Omega_D}\left(3+\Omega_\kappa\right)\right.$$
$$\left.+6b^2\left(\frac{1+\Omega_\kappa}{\Omega_D}\right)-\Omega_D+9\epsilon\left(\Omega_D-\gamma-1\right)\right]^{-\frac{1}{2}}\frac{da}{a}. \quad (66)$$

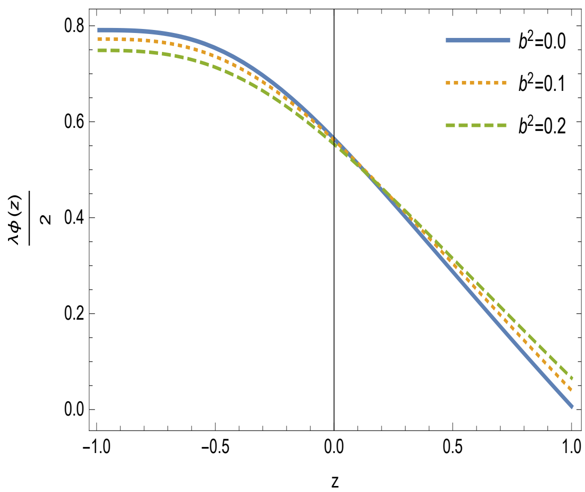

Fig. 21 – Dilaton MGDE scalar field versus redshift for different coupling constants b^2 and viscosity constant $\epsilon = 0$, with auxiliary parameters $\Omega_{k0} = 0.01$, $\Omega_{D0} = 0.76$, $\gamma = 1.105$ and $\Delta G = -0.0025$ based on Eq. (66).

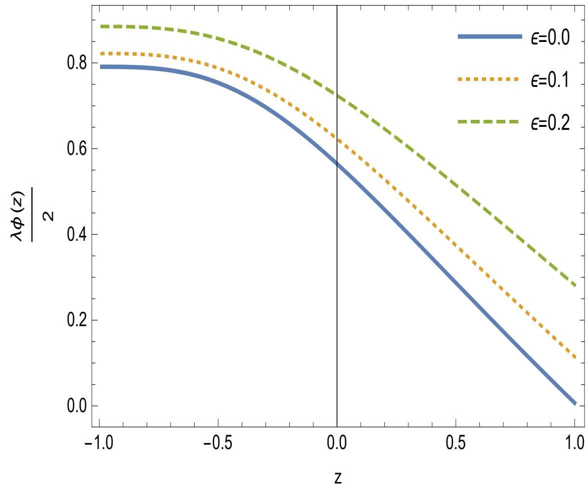

Fig. 22 – Dilaton MGDE scalar field versus redshift for different viscosity constants ϵ and coupling constant $b^2 = 0$, with auxiliary parameters $\Omega_{k0} = 0.01$, $\Omega_{D0} = 0.76$, $\gamma = 1.105$ and $\Delta G = -0.0025$ based on Eq. (66).

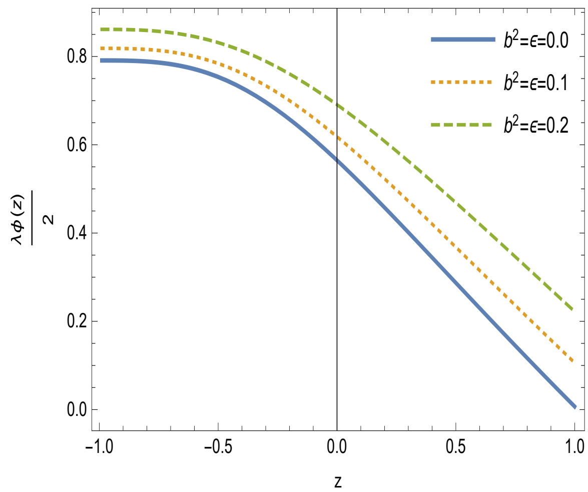

Fig. 23 – Dilaton MGDE scalar field versus redshift for different coupling constants b^2 and viscosity constants ϵ, with auxiliary parameters $\Omega_{k0} = 0.01$, $\Omega_{D0} = 0.76$, $\gamma = 1.105$ and $\Delta G = -0.0025$ based on Eq. (66).

The variation of the dilaton MGDE scalar field, Eq. (66), versus redshift for different coupling constants b^2 and viscosity constants ϵ is plotted in Figures 21 to 23 respectively. From the figures, it is noted that dilaton MGDE scalar field model behaves like the k-essence scalar field model.

4. CONCLUSION

In this paper, we have studied the Veneziano MGDE model in the framework of non-flat FRW universe filled with interacting viscous MGDE and DM in the presence of time varying gravitational constant G. In section 2, we obtained the EoS parameter of interacting viscous MGDE, the deceleration parameter q, as well as the differential equation governing the evolution of the MGDE density parameter and we solved this differential equation numerically. Then we discussed the variation of the MGDE density parameter Ω_D versus redshift z for different coupling constant b^2 in Figure 1.

This Figure 1 shows that:

(i) For a given coupling constant b^2, Ω_D increases during history of the universe in the presence of varying gravitational constant G. It is noted that the evolution of Ω_D is independent of viscosity.

(ii) At the early and late times, Ω_D increases and decreases respectively with the coupling constant b^2. From Figure 2, it is observed that the universe transitions from an early matter dominant epoch to the de-Sitter era in the future, as expected. Similarly, q does not depend on viscosity.

(iii) The EoS parameter ω_D of the MGDE model with varying gravitational constant G and in the absence of viscosity ($\epsilon = 0$), varies from the quintessence phase to the phantom regime. Whereas in the absence of interaction ($b^2 = 0$), it behaves like the freezing quintessence model respectively shown in Figures 3 and 4. It is also noted that the interaction and viscosity have opposite effects on ω_D. Figure 5, for different coupling constants b^2 and viscosity constants ϵ, shows that the interaction and viscosity have opposite effects on ω_D.

In section 3, we established a correspondence between the modified ghost DE energy density with the quintessence, tachyon, k-essence and dilaton energy density in the non-flat FRW universe. We have reconstructed both the dynamics ϕ and potentials $V(\phi)$ of these quintessence, tachyon, k-essence and dilaton scalar field models possessing some significant features of the underlying theory of DE, such as the interacting MGDE model with varying G. It is observed that, in Figures 6 to 17, the quintessence and tachyon MGDE scalar fields in the presence of varying gravitational constant G, for a given b^2 or ϵ, increase during history of the universe. For a given redshift, they decrease and increase with increasing b^2 and ϵ, respectively. The potentials of the quintessence and tachyon MGDE models for a given b^2 or ϵ, decrease with increasing the time. The effects of viscosity and interaction on both the potentials and scalar fields of the aforementioned models, are opposite to each other. It is also observed, from Figures 18 to 23, that the $k-$essence and dilaton MGDE scalar fields for a given b^2 or ϵ increases during the history of the universe.

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REFERENCES

Amendola, L.: 2000, *Phys. Rev. D* **62**, 043511.
Arkani-Hamed, N. et al.: 2004, *J. Cosmol. Astropart. Phys.* **04**, 001.
Armendariz-Picon, C., Damour, T. and Mukhanov, V.: 1999, *Phys. Lett. B* **458**, 209.
Armendariz-Picon, C., Mukhanov, V. and Steinhardt, P. J.: 2000, *Phys. Rev. Lett.* **85**, 4438.
Armendariz-Picon, C., Mukhanov, V. and Steinhardt, P. J.: 2001, *Phys. Rev. D* **63**, 103510.
Banerjee, N. and Pavón, D.: 2007, *Phys. Lett. B* **647**, 477.
Brevik, I.: 2002, *Phys. Rev. D* **65**, 127302.
Brevik, I. and Gorbunova, O.: 2005, *Gen. Relativ. Gravit.* **37**, 2039.
Brevik, I. and Gorbunova, O.: 2008, *Eur. Phys. J. C* **56**, 425.
Brevik, I., Gorbunova, O. and Gomez, D. S.: 2010, *Gen. Relativ. Gravit.* **42**, 1513.
Cai, R. G., Tuo, Z. L., Zhang, H. B. and Su, Q.: 2011, *Phys. Rev. D* **84**, 123501.
Cai, R. G., Tuo, Z. L., Wu, Y. B. and Zhao, Y. Y.:2012 *Phys. Rev. D* **86**, 023511.
Caldwell, R. R., Dave, R. and Steinhardt, P. J .: 1998, *Phys. Rev. Lett.* **80**, 1582.
Caldwell, R. R.: 2002, *Phys. Lett. B* **545**, 23.
Caldwell, R. R. and Linder, E. V.: 2005, *Phys. Rev. Lett.* **95**, 141301.
Chiba, T., Okabe, T. and Yamaguchi, M.: 2000, *Phys. Rev. D* **62**, 023511.
Copeland, E. J., Sami, M. and Tsujikawa, S.: 2006, *Int. J. Mod. Phys. D* **15**, 1753.
Ebrahimi, E. and Sheykhi, A.: 2011, *Phys. Lett. B* **705**, 19.
Ebrahimi, E. and Sheykhi, A.: 2011, *Int. J. Mod. Phys. D* **20**, 2369.
Eckart, C.: 1940, *Phys. Rev.* **58**, 267.
Elizalde, E., Nojiri, S., Odintsov, S. D. and Wang, P.: 2005, *Phys. Rev. D* **71**, 103504.
Garriga, J. and Mukhanov, V.: 1999, *Phys. Lett. B* **458**, 219.
Gasperini, M., Piazza, F. and Veneziano, G.: 2002, *Phys. Rev. D* **65**, 023508.
Granda, L. N. and Oliveros, A.: 2009, *Phys. Lett. B* **671**, 199.
Forbes, M. M. and Zhitnitsky, A. R.: 2008, *Phys. Rev. D* **78**, 083505.
Holdom, B.: 2011 *Phys. Lett. B* **697**, 351.
Ichiki, K. et. al.: 2008, *JCAP* **0806**, 005.
Ishida, E. E. O. et al.: 2008, *Astropart. Phys.* **28**, 547.
Jamil, M., Saridakis, E. N., Setare, M. R.:2009 *Physics Letters B* **679**, 172-176.
Jamil, M. and Farooq, M. U.: 2010, *J. Cosmol. Astropart. Phys.* **03**, 001.
Jawad, A. and Majeed, A.: 2015, *Astrophys. Space Sci.* **356**, 375.
Karami, K. and Fehri, J.: 2010, *Phys. Lett. B* **684**, 61.
Karami, K. and Abdolmaleki, A.: 2012, *J. Cosmol. Astropart. Phys.* **04**, 007.
Karami, K. et al.: 2013, *Int. J. Mod. Phys. D* **22**, 1350018.
Kawarabayashi, K, and Ohta, N.:1980, *Nucl. Phys. B* **175**, 477.
Khodam-Mohammadi, A. et al.: 2012, *Mod. Phys. Lett. A* **27**, 1250100.
Maggiore, M.: 2011a, *Phys. Rev. D* **83**, 063514.
Maggiore, M. et al.: 2011b, *Phys. Lett. B* **704**, 102.
Nath, P. and Arnowitt, R. L.: 1981, *Phys. Rev. D* **23**, 473.
Ohta, N.: 2011, *Phys. Lett. B* **695**, 41.
Padmanabhan, T.: 2002, *Phys. Rev. D* **66**, 021301.
Padmanabhan, T. and Choudhury, T. R.: 2002, *Phys. Rev. D* **66**, 081301.

Pavon, D. and Zimdahl, W.: 2005, *Phys. Lett. B* **628**, 206.
Perlmutter, S. et al.:1997, *Astrophys. J.* **483**, 565.
Perlmutter, S. et al.: 1998, *Nature* **391**, 51.
Perlmutter, S. et al.: 1999, *Astrophys. J.* **517**, 565.
Ratra, B. and Peebles, J.: 1998, *Phys. Rev. D* **37**, 321.
Riess, A. G. et al.: 2004, *Astrophys. J.* **607**.
Riess, A. G. et al.: 2007, *Astrophys. J.* **659**, 98.
Rosenzweig, C., Schechter, J. and Trahern, C. G.: 1980, *Phys. Rev. D* **21**, 3388.
Rozas-Fernandez, A.: 2012, *Phys. Lett. B* **709**, 313.
Rozas-Fernandez, A., Brizuela, D. and Cruz, N.: 2010, *Int. J. Mod. Phys. D* **19**, 573.
Rozas-Fernandez, A.: 2011, *Eur. Phys. J. C* **71**, 1536.
Sen, A.: 1999 *J. High Energy Phys.* **10**, 008.
Sen, A.: 2002a, *J. High Energy Phys.* **04**, 048.
Sen, A.: 2002b, *J. High Energy Phys.* **07**, 065.
Setare, M. R.: 2007, *Phys. Lett. B* **653**, 116.
Sheykhi, A.: 2011, *Phys. Scr.* **84**, 065904.
Sheykhi, A.: 2009, *Int. J. Mod. Phys. D* **18**, 2023.
Sheykhi, A. and Bagheri, A.: 2011, *Europhys. Lett.* **95**, 39001.
Sheykhi, A. and Movahed, M. S.: 2012a, *Gen. Relativ. Gravit.* **44**, 449.
Sheykhi, A., Movahed, M. S. and Ebrahimi, E.: 2012b, *Astrophys. Space Sci.* **339**, 93.
Thomas, E., Zhitnitsky, A. R.: 2012, *Phys. Rev. D* **85**, 044039.
Urban, F. R. and Zhitnitsky, A. R.: 2009, *Phys. Rev. D* **80** 063001.
Urban, F. R. and Zhitnitsky, A. R.: 2010, *Phys. Lett. B* **688**, 9.
Veneziano, G.: 1979, *Nucl. Phys. B* **159**, 213.
Weinberg, S.: 1972, Gravitation and Cosmology, (John Wiley & Sons), Singapore, ASIA.
Weinberg, S.: 1989, *Rev. Mod. Phys.* **61**, 1.
Witten, E.: 1979, *Nucl. Phys. B* **156**, 269.
Wu, J. P., Ma, D. J. and Ling, Y.: 2008, *Phys. Lett. B* **663**, 152.
Zhang, H. and Zhu, Z.: 2006, *Phys. Rev. D* **73**, 0435189.
Zhang, X.: 2007, *Phys. Lett. B* **648**, 1.
Zhang, J., Zhang, X. and Liu, H.: 2007, *Phys. Lett. B* **651**, 84.
Zhitnitsky, A. R.: 2010, *Phys. Rev. D* **82**, 103520.
Zhitnitsky, A. R.: 2012, *Phys. Rev. D* **86**, 045026.
Zimdahl, W. and Pavon, D.: 2001, *Phys. Lett. B* **521**, 133.

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