ON THE DETERMINATION OF THE ORBITAL ELEMENTS IN THE LIGHT TRAVEL TIME EFFECT

ALEXANDRU POP[1], MARIA CRĂCIUN[2], MIHAI BĂRBOSU[3,1]

[1]*Astronomical Institute, Astronomical Observatory Cluj-Napoca, Romanian Academy,Cireşilor 19, Cluj-Napoca, 400487, Romania Email: andi_pop@yahoo.com*
[2]*Tiberiu Popoviciu Institute of Numerical Analysis, Romanian Academy, Fântânele 57, Cluj-Napoca, 400110, Romania Email: craciun@ictp.acad.ro*
[3]*Rochester Institute of Technology, School of Mathematical Sciences, New York, USA Email: mxbsma@rit.edu*

Abstract. This paper deals with the light travel time effect in binary systems with variable components. The methods for estimation of the orbital elements relying on the parameters obtained through Fourier decomposition of the respective timing / $O-C$ data, are reviewed. Some new formulae are derived and some qualitative remarks on the adopted methodology are discussed. Finally, some issues related to the presence of time gaps in the analysed data and to the case of noisy data are addressed.

Key words: Variable stars – Binary systems – Data analysis.

1. INTRODUCTION

Unseen companions of variable stars (either pulsating stars or eclipsing close binary systems, or planets around pulsars) can be detected through the well-known *Light Travel Time Effect* (LTTE). It consists of the modulation of the variable star period induced by the orbital motion around the barycentre of the system (variable star + unseen companion(s)). As a consequence, the occurrence of the corresponding extremum light times is also modulated. The study of the LTTE has been tackled by different authors, like Woltjer (1922), De Sitter (1933), Irwin (1952, 1959), Kopal (1959, 1978) and Borkovits & Hegedüs (1996), Konacki & Maciejewski (1996, 1999), Pop (1998, 1999, 2000). The common goal of their approaches was to estimate the values of the orbital elements for the corresponding system.

Being given a timing data set and choosing the mathematical model which rules the LTTE, the values of the orbital elements are frequently estimated through the Levenberg-Marquardt least-squares minimization method. However, its application requires first/initial guesses of the values of the orbital elements. When the complexity of the investigated period variability phenomenon is relatively high, the identification of the set of initial values which leads towards the optimal fit, can be a real challenge. The solution to this problem, which was adopted by several authors, was to explore a large number of initial guesses (*e.g.*, Hinse *et al.*, 2012a, 2014b; Lee *et al.*, 2014; Borkovits *et al.*, 2015; Esmer *et al.*, 2021).

The aim of the present paper is to review the problem of the estimation of first guesses of orbital elements relying on the parameters of an ephemeris containing a truncated Fourier series (*e.g.*, Pop 1996) and taking into account the Fourier decomposition of the LTTE (see Kopal 1959, 1978; Borkovits & Hegedüs 1996; Konacki & Maciejewski 1996; Pop 1998, 1999, 2000). Some new formulae are derived and different practical aspects are also considered in order to account for the presence of time gaps in the analysed data as well as for the presence of the additive observational noise.

2. THE LTTE MODEL

The occurrence of the extremum (minimum or maximum) light times of a variable star which has an unseen companion (stellar or planetary) is described by the well-known ephemeris (*e.g.*, Kopal 1978)

$$t_n = t_0 + P_p n + \frac{r_n}{c} \sin i \sin(\nu_n + \omega), \tag{1}$$

where t_0 is the initial epoch, P_p is the period of the observed variable star, and n is the variability cycle number. The last term represents the time delay required for the light to travel across the projection of the radius vector r_n of the variable star orbit onto the line of sight. In the formulae given below, we used the following notations: $a \sin i$ is the projected semi-major axis of the absolute orbit of the barycentre of the variable component around the barycentre of the hypothetical binary or triple system, i is the inclination of the normal to the orbital plane with respect to the line of sight, c is the speed of light, e is the orbital eccentricity, ω is the periastron longitude, T_p is the time of periastron passage, and P_s is the orbital period.

When assuming more unseen companions, the LTTE model can also be applied, but only by neglecting the gravitational perturbations between companions (*e.g.*, Hinse *et al.*, 2012b, 2014a).

Taking into account the well-known expansions of the elliptic motion (*e.g.*, Brouwer & Clemence 1961; Townsend & Tamburro 1968; Kopal 1978), we have

$$\frac{r_n}{a} \cos \nu_n = F_0(e) + \sum_{k=1}^{\infty} F_k(e) \cos(k M_n), \tag{2}$$

$$\frac{r_n}{a} \sin \nu_n = \sum_{k=1}^{\infty} G_k(e) \sin(k M_n), \tag{3}$$

with

$$F_0(e) = -\frac{3}{2} e, \quad F_k(e) = \frac{2}{k} J'_k(ke), \quad G_k(e) = 2\sqrt{1-e^2}\,\frac{J_k(ke)}{ke}, \ k = 1, 2, \dots . \tag{4}$$

Thus, Equation (1) becomes

$$t_n = t_0 + \tau_0 F_0(e)\sin\omega + P_p n + \\ + \sum_{k=1}^{\infty} \tau_0 [G_k(e)\cos\omega \sin(kM_n) + F_k(e)\sin\omega\cos(kM_n)], \quad (5)$$

where

$$\tau_0 = \frac{a \sin i}{c}, \quad (6)$$

$$kM_n = 2\pi k f_0 n + k\Phi_0, \quad (7)$$

$$\Phi_0 = \Omega_s(t_0 - T_p), \quad \Omega_s = \frac{2\pi}{P_s}, \quad (8)$$

$$f_0 = \frac{P_p}{P_s}, \quad (9)$$

M_n and f_0 being the mean anomaly and the dimensionless frequency which features the periodic behaviour of the $O-C$ curve respectively, while Ω_s is the orbital angular frequency or the mean motion. This way, the LTTE ephemeris (Equation (1)) becomes

$$t_n = t_0 + \tau_0 F_0(e)\sin\omega + P_p n + \\ + \sum_{k=1}^{\infty} \tau_0 [G_k(e)\cos\omega \sin(2\pi k f_0 n + k\Phi_0) + \\ + F_k(e)\sin\omega\cos(2\pi k f_0 n + k\Phi_0)]. \quad (10)$$

Kopal (1978) rewrote it in the form

$$t_n = t_0 + \tau_0 F_0(e)\sin\omega + P_p n + \\ + \sum_{k=1}^{\infty} \tau_0 [a_k \sin(2\pi k f_0 n) + b_k \cos(2\pi k f_0 n)], \quad (11)$$

with the following notations

$$a_k = \tau_0 [G_k(e)\cos\omega\cos(k\Phi_0) - F_k(e)\sin\omega\sin(k\Phi_0)], \quad (12)$$

$$b_k = \tau_0 [G_k(e)\cos\omega\sin(k\Phi_0) + F_k(e)\sin\omega\cos(k\Phi_0)]. \quad (13)$$

From Eqs. (10) and (11) it immediately results

$$a_k^2 + b_k^2 = \tau_0^2 [G_k^2(e)\cos^2\omega + F_k^2(e)\sin^2\omega] = \tau_0^2 A_k^2 = \tau_k^2. \quad (14)$$

Pop (1998, 1999, 2000) used slightly different notations in Equation (10):

$$a_k = G_k(e)\cos\omega = A_k \cos\varphi_k, \quad (15)$$

$$b_k = F_k(e)\sin\omega = A_k \sin\varphi_k, \quad (16)$$

with

$$\frac{b_k}{a_k} = \frac{F_k(e)}{G_k(e)} \tan\omega = \tan\varphi_k, \qquad (17)$$

to write the LTTE ephemeris in the form

$$t_n = t_0 + \tau_0 F_0(e)\sin\omega + P_p n + \sum_{k=1}^{\infty} \tau_k \sin(2\pi k f_0 n + \Phi_k), \qquad (18)$$

where

$$\tau_k = \tau_0 A_k, \quad A_k = \sqrt{a_k^2 + b_k^2} = \sqrt{G_k^2(e)\cos^2\omega + F_k^2(e)\sin^2\omega}, \qquad (19)$$

$$\Phi_k = \varphi_k + k\Phi_0, \quad k = 1, 2, \ldots\ . \qquad (20)$$

From Equation (18) one immediately can see that the times of extremum light displayed by a variable star having an unseen companion are modulated by an *arbitrary periodic signal*, its parameters being functions of the orbital elements of the system, which is a previously known result. Consequently, the *amplitude spectrum of the $O-C$ residuals* computed from these extremum light times is a *line spectrum*, consisting of an infinite number of lines situated at the frequencies $f_{0k} = kf_0$ and having the monotonically decreasing amplitudes τ_k $(k = 1, 2, \ldots)$ [see also Konacki & Maciejewski (1996, 1999)].This characteristic feature is useful, *e.g.*, in the case of Algol type systems, in the situation where we need to make a differential diagnosis with respect to the quasiperiodic modulation induced by the magnetic cyclic activity occurring in the cool, late-spectral type secondary component (*e.g.*, Pop *et al.*., 2017).

The constant term $\tau_0 F_0(e)\sin\omega$ which appears in Equation (18) is actually included in the observed times of minimum light. In fact, within the frame of our approach, the estimation of initial guesses of the orbital elements relies on the equivalence between the LTTE term in Equation (1) and the truncated Fourier series appearing in Eqs. (11) and (18) [see also Hinse *et al.*, (2012a)].

Note that the LTTE ephemeris, as derived by Kopal (1978) (Equation (10)), can be written as Equation (18), but with the following notation [see Equations (12) and (13)]:

$$\tan\Phi_k = \frac{b_k}{a_k} = \frac{G_k(e)\cos\omega\sin(k\Phi_0) + F_k(e)\sin\omega\cos(k\Phi_0)}{G_k(e)\cos\omega\cos(k\Phi_0) - F_k(e)\sin\omega\sin(k\Phi_0)}. \qquad (21)$$

3. FIRST GUESSES OF THE ORBITAL ELEMENTS

In this section we present some methods for estimating the values of the orbital elements of the (hypothetical) absolute orbit of the variable star around the system's barycentre; these elements are: τ_0 or $a\sin i$, e, ω, P_s, and T_p. The value of the orbital period can be immediately calculated from the formula $P_s = P_p/f_0$, in which, the values of P_p and f_0 resulted from the fitting of the model (11) or (18) to the observational data.

Kopal's (1978) approach kept the first **two** periodic terms from the truncated Fourier series appearing in the LTTE ephemeris (Equation (11)) and after that, formulae for the orbital elements were derived. He also made some approximations in the formulae of the $F_k(e)$ and $G_k(e)$ coefficients:

$$F_1(e) = G_1(e) \cong 1, \quad F_2(e) = G_2 \cong \frac{e}{2}. \tag{22}$$

Therefore, the orbital elements became:

$$\tau_0 = \sqrt{a_1^2 + b_1^2}, \tag{23}$$

$$e = 2\sqrt{\frac{a_2^2 + b_2^2}{a_1^2 + b_1^2}}, \tag{24}$$

$$\omega = \tan^{-1}\left[\frac{(b_1^2 - a_1^2)b_2 + 2a_1a_2b_1}{(a_1^2 - b_1^2)a_2 + 2a_1b_1b_2}\right], \tag{25}$$

$$T_p = t_0 - \frac{1}{\Omega_s}\tan^{-1}\left[\frac{a_1b_2 - b_1a_2}{a_1a_2 + b_1b_2}\right]. \tag{26}$$

Tests performed by Borkovits & Hegedüs (1996) indicated that reliable results could be obtained with these formulae for $e < 0.6$.

Borkovits & Hegedüs (1996) also derived corresponding formulae for the case in which the first **three** periodic terms in Equation (11) were considered and they also took into account second order approximations in order to estimate the orbital elements. Thus, these approximations for the coefficients $F_k(e)$ and $G_k(e)$ are [see also, *e.g.*, Brouwer & Clemence (1961)]:

$$F_1(e) \cong 1 - \frac{3e^2}{8}, \quad G_1(e) \cong 1 - \frac{5e^2}{8}, \tag{27}$$

$$F_2(e) = G_2(e) \cong \frac{e}{2}, \tag{28}$$

$$F_3(e) = G_3(e) \cong \frac{3e^2}{8}. \tag{29}$$

The corresponding formulae given by Borkovits & Hegedüs (1996) for the orbital elements become (with our notations)

$$e = \frac{4}{3}\sqrt{\frac{a_3^2 + b_3^2}{a_2^2 + b_2^2}}, \tag{30}$$

$$\omega = \tan^{-1}\left[\frac{(b_1^2 - a_1^2)b_2 + 2a_1a_2b_1}{(a_1^2 - b_1^2)a_2 + 2a_1b_1b_2}\left(1 - \frac{e^2}{3}\right)\right], \tag{31}$$

$$T_p = t_0 - \frac{1}{\Omega_s} \tan^{-1} \left[\frac{1 - \dfrac{a_1 F_1(e)}{b_1 G_1(e)} \tan\omega}{\dfrac{a_1}{b_1} + \dfrac{F_1(e)}{G_1(e)} \tan\omega} \right], \tag{32}$$

$$\tau_0 = \frac{a_1^2 + b_1^2}{F_1^2(e) + [G_1^2(e) - F_1^2(e)] \cos^2\omega}. \tag{33}$$

The approach of Pop (1998, 1999, 2000) relied on the LTTE ephemeris given by Equation (18). Its aim was to develop an algorithm for the estimation of the orbital elements by:

(i) taking into account an **arbitrary** number (K) of amplitudes in a truncated Fourier series of type (18);

(ii) using higher accuracy estimates of the $F_k(e)$ and $G_k(e)$ coefficients, which rely on Bessel functions [Equation (4); see also Konaki & Maciejewski (1996)] or on the equivalent power series in e (Pop 1998, 2000).

The estimation of the values of e and ω relies on the remark that amplitude ratios are functions depending only on the orbital elements [see also Kopal (1978) and Konaki & Maciejewski (1996, 1999)]

$$R_{k1}^{calc} = \frac{\tau_k}{\tau_1} = \frac{A_k(e,\omega)}{A_1(e,\omega)}, \quad k = 2, 3, \ldots, K. \tag{34}$$

Let us denote $R_{k1}^{obs} = \tau_k^{obs}/\tau_1^{obs}$, the respective amplitudes being those estimated from the Fourier decomposition of the 'observed' $O - C$ curve or by fitting one of the models (11) or (18) to the analysed timing data. As proposed by Pop (1998, 1999, 2000), the values of e and ω can be estimated by minimizing the objective function

$$\delta = \sqrt{\sum_{k=2}^{K} [R_{k1}^{obs} - R_{k1}^{calc}(e,\omega)]^2}, \tag{35}$$

using, *e.g.*, a simple algorithm of exhaustive search. Note that the above amplitude ratios were considered by Pop by analogy with the structural parameters of the light curves of pulsating stars, defined on the basis of their Fourier amplitudes (*e.g.*, Simon & Lee 1981).

We note that:

(a) due to the fact that A_k $(k = 1, 2, \ldots, K)$ are periodic functions with a period of π, the ambiguity between the values ω and $\omega + \pi$ has to be removed;

(b) according to our numerical tests, the objective function δ [see Equation (35)] is much more sensitive to the variation of e compared to the one with respect to ω (see also Figs. 4 and 5 below). Therefore, we proposed another approach for estimating the value of ω [see Equation (39) below].

The value of τ_0 can be estimated from Eqs. (19) above. Thus,

$$\tau_0 \cong \frac{\tau_1^{obs}}{A_1(e,\omega)}, \quad A_1(e,\omega) = \sqrt{G_1^2(e)\cos^2\omega + F_1^2(e)\sin^2\omega}. \tag{36}$$

According to the initial approach of Pop (1998, 1999, 2000), the value of T_p was estimated by identifying – using the Fourier series in Equation (18) – the cycle number (n_p) corresponding to the extremum light time which is the closest to the time of periastron passage and which is contained within a time interval of one orbital period P_s (or P_0, if we expressed the time in cycle numbers), centred on the initial epoch t_0 (n_0). The corresponding deviation from the linear ephemeris is

$$\tau_{0p} = \tau_0 \sin\omega, \tag{37}$$

while

$$T_p = t_0 + P_p n_p. \tag{38}$$

Within the present study we provide additional formulae for estimating the values of ω and T_p. Thus, in the case of the LTTE ephemeris (11) used by Kopal (1978) and Borkovits & Hegedüs (1996), we derived the following third-degree equation in $\tan\omega$:

$$\frac{b_2(b_1^2-a_1^2)+2a_1b_1a_2}{a_2(a_1^2-b_1^2)+2a_1b_1b_2} = \frac{G_1(e)[2F_1(e)G_2(e)-G_1(e)F_2(e)]+F_1^2(e)F_2(e)\tan^2\omega}{G_1^2(e)G_2(e)+F_1(e)[2G_1(e)F_2(e)-F_1(e)G_2(e)]\tan^2\omega}\tan\omega. \tag{39}$$

This method may be applied after a preliminary estimation of e and ω through the minimization of δ (Equation (35)). The obtained value of e is then introduced in Equation (39) which can be solved numerically.

In the case of the LTTE ephemeris (18) used by Pop (1998, 1999, 2000), taking into account Eqs. (19), (17), and (8), we derived the following formula for T_p:

$$T_p = t_0 - \frac{1}{\Omega_s}\left[\phi_1 - \tan^{-1}\left[\frac{F_1(e)}{G_1(e)}\tan\omega\right]\right], \tag{40}$$

which is equivalent to Equation (32). As it can be seen from equations (36), (37), and (38), but also (39) and (40), precise determinations of e and ω are very important in the computation of τ_0 and T_p. That is why, in Section 4 below, we will illustrate the estimation of these two orbital parameters.

4. QUALITATIVE REMARKS ON AMPLITUDE RATIOS AND THE OBJECTIVE FUNCTION

Regarding the amplitude ratios from Equation (34), R_{21}^{calc}, R_{31}^{calc}, R_{41}^{calc}, and the objective function, δ, from Equation (35), we add the following remarks.

In Figure 1 we plotted the ratios R_{21}, R_{31}, and R_{41} (in descending order) for all values of e and ω [$e \in (0,1)$, $\omega \in [0,\pi)$]. One can observe that $R_{k1}(e,\omega) =$

$R_{k1}(e, \pi - \omega)$, $k = 2, 3, \ldots, K$, therefore the graphical representations of R_{k1} are symmetric with respect to the plane $\omega = \pi/2$. Konacki & Maciejewski (1996, 1999) had a similar plot approach, for two of these quantities, stating that these ratios are, "*for small eccentricities (...), with good accuracy, functions of* e *only*", remark similar to the one above in (b). We can add that Figure 1 shows the surfaces of R_{21}, R_{31}, and R_{41} related to the LTTE induced by a purely Keplerian orbital motion.

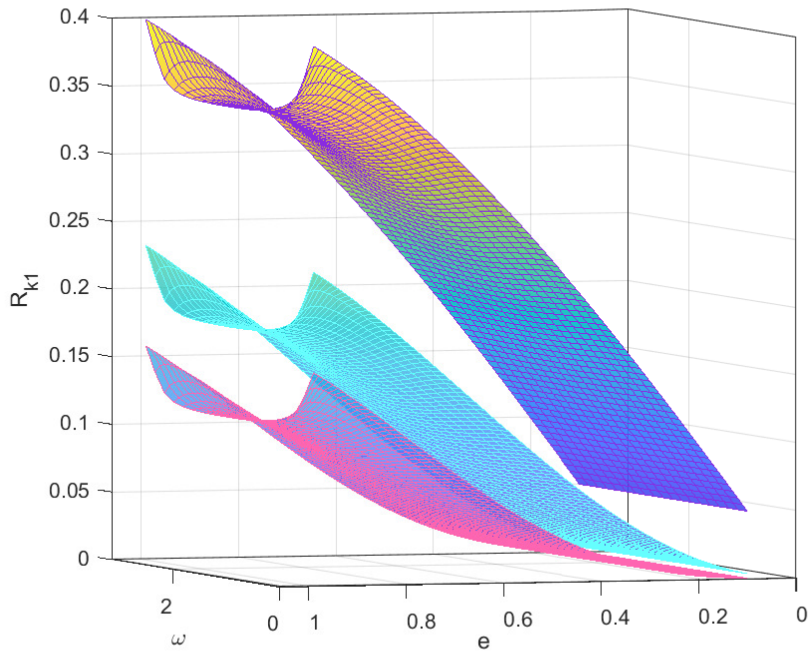

Fig. 1 – Amplitude ratios R_{21}, R_{31}, and R_{41} as functions of e and ω.

Figure 2 displays the locus of the points of coordinates (R_{21}, R_{31}, R_{41}), for all e and ω, corresponding to a purely Keplerian LTTE. It can be used in differentiating the LTTE mechanism when compared to other arbitrary periodic phenomena and in determining how close a hypothetical movement is to a Keplerian one (see also Figure 1 of Pop, 1999).

In order to illustrate our approach, we generated a data set of LTTE timing data using the orbital elements for the hypothetical companion of the W UMa type binary system V524 Monocerotis (He *et al.* 2012; Bulut & Aşkın 2022), superposed on a Gaussian noise with a relatively low standard deviation (see Figure 3). Due to the fact that the number of the available observed data is relatively small (66), and the orbital cycle coverage is not rich, we increased the number of artificially generated data

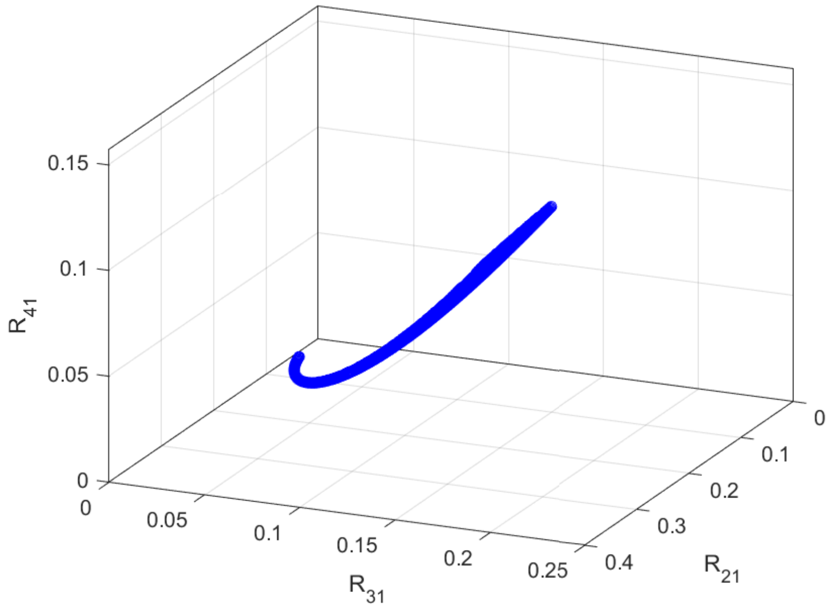

Fig. 2 – Amplitude ratios (R_{21}, R_{31}, R_{41}) for LTTE, for all values e and ω.

(200), keeping the same time base. In addition, the data sampling is also irregular and contains random length gaps.

In Figure 4 we plotted the corresponding objective function, δ, as given in Equation (35), for the amplitude ratios R_{k1}^{obs} for the simulated data set.

Figure 5 shows the contour plot of the isolines of the objective function given in Equation (35) for the simulated LTTE data. As we already mentioned above, for small values of e, the objective function δ is less sensitive to the variation of ω, while for larger values of e, δ becomes more and more sensitive with respect to ω (see also Figure 4).

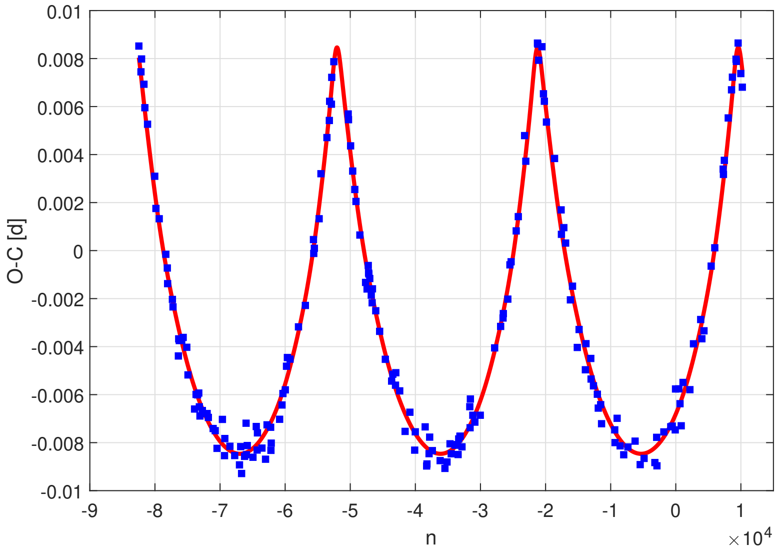

Fig. 3 – $O - C$ diagram of synthetic timing data.

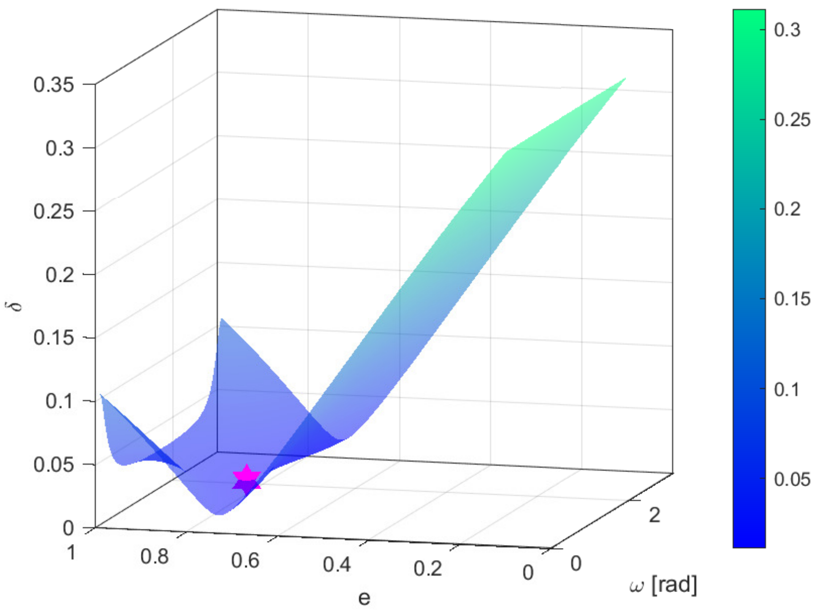

Fig. 4 – Objective function δ for all e and ω.

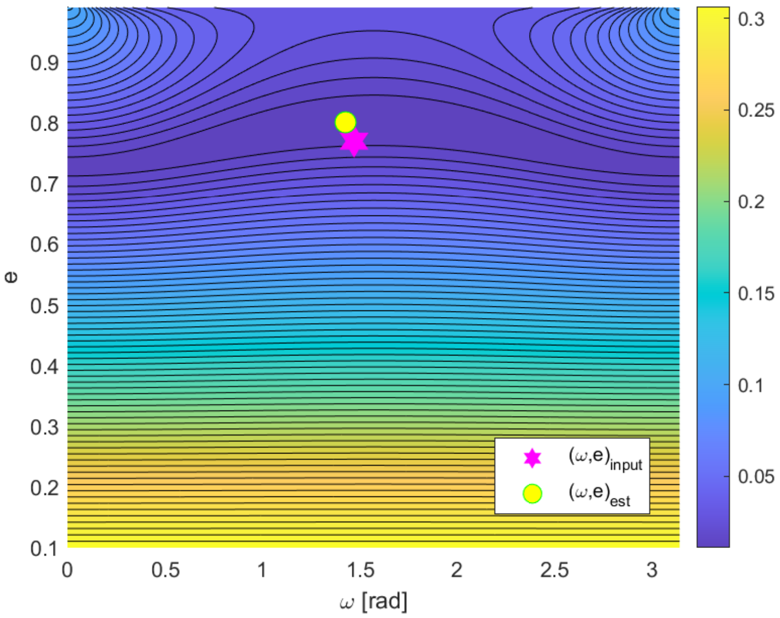

Fig. 5 – Contour plot of the isolines of the objective function δ.

5. PRACTICAL ASPECTS

As previously indicated, in order to consider the investigation of a period variability phenomenon possibly caused by a LTTE, we have to fit a truncated Fourier series to the available timing data. Therefore, it is useful to take into account the run of the respective $O-C$ curve, and also to analyse its amplitude spectrum in order to obtain further information. The following situations may be encountered:

(i) Most frequently the timing data displays time gaps which limits the order of the fitted Fourier series. Thus, according to (a) the local weather conditions experienced by the respective observers, and, sometimes to (b) the historical context (see, *e.g.*, the case of the two World Wars), the available timing data can contain more or less wide time gaps. In order to avoid the oscillation of the fitted curve inside these gaps, a simple condition has to be fulfilled [similar to that proposed by Pop et al. (2004)]: half of the minimum dimensionless period P_{0min} $(P_0 = 1/f_0 = P_s/P_p)$ corresponding to the frequency of the highest harmonic which will be taken into account $K_{max}f_0$ has to be equal to the width of the largest time gap measured in cycle numbers (Δn_{max}), *i.e.*,

$$P_{0min}/2 = \Delta n_{max}, \quad (41)$$

which becomes

$$K_{max} = 1/(2f_0\Delta n_{max}). \tag{42}$$

(ii) According to the observational technique and its location (ground or space based), the analysed timing data may contain a more or less amount of additive observational noise.

(iii) In the case of the Fourier expansion of the elliptical motion, the large orbital eccentricity values yield large amplitudes of the higher order harmonics (see Konacki & Maciejewski 1996). On the other hand, according to the level of the observational noise, only (very) low order harmonics can be observed in the amplitude spectrum of the detrended $O-C$ residuals. In case of lower eccentricity orbits, the second harmonic $(3f_0)$ and even the first one $(2f_0)$ may be significantly contaminated by the contribution of the observational noise. As a consequence, by taking too many harmonics, the fitted model will tend to describe the fluctuations caused by the noise. Clearly, in such a situation, the attempt to estimate the values of the orbital elements might be seriously affected.

An elegant solution to this problem may be the use of the Lanczos smoothing factors, already applied by Meylan & Burki (1986) in the case of the radial velocity curves of some pulsating stars. The Lanczos factors are in fact weights which multiply the estimated amplitudes of a truncated Fourier series [see, *e.g.*, Arfken (1970), Hamming (1986)]. For our model, in Equation (18) they are given by the formula

$$\sigma_k(K) = \frac{\sin\dfrac{\pi k}{K}}{\dfrac{\pi k}{K}}, \quad (k = 1, 2, \ldots, K). \tag{43}$$

As it is well-known, these factors have values less than 1 and, for $k = K$, $\sigma_K(K) = 0$, which means that the highest harmonic taken into account is completely eliminated. Thus, by reducing the harmonics amplitudes as well as their number, the Lanczos factors perform a smoothing of the Fourier fit. Therefore, a predictable consequence might be an underestimation of the orbital eccentricity value.

REFERENCES

Arfken, G. 1970, *Mathematical Methods for Physicists*, Second Edition, Academic Press, New York

Borkovits, T., Hegedüs, T.: 1996, *Astron. Astrophys. Suppl.* , **120**, 63

Borkovits, T., Rappaport, S., Hajdu, T., Sztakovics, J.: 2015, *Mon. Not. Roy. Astron. Soc.* , **448**, 946

Brouwer, D., Clemence, G.M.: 1961, *Methods in Celestial Mechanics*, Academic Press, New York and London

Bulut, İ., Aşkın, G.: 2022, *New. Astron.*, **90**, 101668 (12 pp.)

Burki, G., Meylan, G.: 1986, *Astron. Astrophys.* , **156**, 131

De Sitter, A.: 1933, *Bull. Astron. Inst. Netherlands*, **7**, 119

Esmer, E.M., Baştürk, Ö., Hinse, T.C., Selam, S.O., Correia, A.C.M.: 2021, *Astron. Astrophys.* , **648**, id. A85, (22 pp.)

Hamming, R.W.: 1986, *Numerical Methods for Scientists and Engineers*, Second Edition, Dover Publications, Inc., New York
He, J.-J., Wang, J.-J., Qian, S.-B.: 2012, *Pub. Astron. Soc. Japan* , **64**, 85
Hinse, T.C., Horner, J., Wittenmyer, R. A.: 2014a, *J. Astron. Space. Sci.*, **31**, 187
Hinse, T.C., Goździewski, K., Lee, J.W., Haghighipour, N., Lee, C.-U.: 2012a, *Astron. J.* , **144**, 34 (10 pp.)
Hinse, T.C., Lee, J. W., Goździewski, K., Haghighipour, N., Lee, C.-U., Scullion, E. M.: 2012b, *Mon. Not. Roy. Astron. Soc.* , **420**, 3609
Hinse, T.C., Horner, J., Lee, J.W., Wittenmyer, R.A., Lee, C.-U., Park, J.-H., Marshall, J.P.: 2014b, *Astron. Astrophys.* , **565**, A104 (8 pp.)
Irwin, J.B.: 1952, *Astrophys. J.* , **116**, 211
Irwin, J.B.: 1959, *Astron. J.* , **64**, 149
Konacki, M., Maciejewski, A.J.: 1996, *Icarus*, **122**, 347
Konacki, M., Maciejewski, A.J.: 1999, *Mon. Not. Roy. Astron. Soc.* , **308**, 167
Kopal, Z.: 1959, *Close Binary Systems*, Chapman-Hall and John Wiley, London and New York
Kopal, Z.: 1978, *Dynamics of Close Binary Systems*, D. Reidel Publishing Company, Dordrecht
Lee, J.W., Hong, K., Hinse, T.C.: 2015, *Astron. J.* , **149**:93 (7 pp.)
Lee, J.W., Hinse, T.C., Youn, J.-H., Han, W.: 2014, *Mon. Not. Roy. Astron. Soc.* , **445**, 2331
Pop, A.: 1996, *Romanian Astron. J.* , **6**, 147
Pop, A.: 1998, *PhD Thesis*, Astronomical Institute of the Romanian Academy, Bucharest
Pop, A.: 1999, *Romanian Astron. J.* , **9**, 131
Pop, A.: 2000, *Metode de analiză şi interpretare a variabilităţii stelare*, Casa Cărţii de Ştiinţă, Cluj-Napoca
Pop, A., Turcu, V., Codreanu, S.: 2004, *Astrophys. Space Sci.* , **293**, 393
Pop, A., Crăciun, M., Mocanu, G., Vamoş, C.: 2017, *Astrophys. Space Sci.* , **362**, 76
Simon, N.R., Lee, A.S.: 1981, *Astrophys. J.* , **248**, 291
Townsend, G.E., Tamburro, M.B.: 1968, *Guidance, Flight Mechanics and Trajectory Optimization*, Vol. III, *The Two-Body Problem*, NASA CR-1002, Washington, D.C.
Woltjer, J.: 1922, *Bull. Astron. Inst. Netherlands*, **1**, 93

Received on 21 December 2021