# SOME KINEMATICAL PROPERTIES OF SOLAR MOTION USING STELLAR ASSOCIATIONS 

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#### Abstract

In this paper, we have constructed many models of the Solar motion and the kinematical properties for a sample of 5100 main sequence stars; into which 1528 early types (B5, A0, A5, F0, and F5) and about 3572 late types (G0, G5, K0, and K5). The sample was collected with aid of Extended Hipparcos Compilation (XHIP) Cross references. Based on the space velocity vectors ( $\mathrm{U}, \mathrm{V}, \mathrm{W}$ ), we have computed velocity ellipsoid parameters of the stellar associations under study. For both early and late types, we have derived the following constants: the Galactic longitude $l_{A}$, Galactic latitude $b_{A}$, the Solar apex $\left(\alpha_{A}, \delta_{A}\right)$, the Solar velocity $S_{\odot}$, the velocity dispersion $\left(\sigma_{1}, \sigma_{2}, \sigma_{3}\right)$, the distance to the Galactic center $\mathrm{R}_{g c}$, and the projected distances ( $\mathrm{X}_{\odot}, \mathrm{Y}_{\odot}, \mathrm{Z}_{\odot}$ ).


Key words: Stellar associations - Solar elements - Spectral classes - Kinematical analysis - Oort's constants.

## 1. INTRODUCTION

It is undoubtedly true that the different correlations between the kinematical and the physical properties of stars in our Galaxy are, in fact, the local points of Galactic research. As for examples, the variation of kinematic properties with stellar age has been established early (Parenago 1950). Some of the most important results that have emerged from studies of the high-velocity stars are the starting correlation that exists between their spectrophotometric and kinematical properties (Eggen, 1986).

Many authors, like Perrine (1916 and 1917), Pasetto et al. (2012), Ralph Schönrich (2012), and Khyati and Rodrigo (2017), have investigated the Solar motion parameters, like Solar apex, Solar velocity, etc., with different methods and different input data.

Cudworth (1974) and Mihalas \& Binney (1981) considered planetary nebulae as a stellar system for the determination of the Solar apex position and the speed of
the Solar motion.
Using the radial velocity, proper motion, and a space velocity of about 5700 stars from the Bright Star Catalog, Jaschek and Valbouswuet (1991, 1992, 1993, and 1994) determined the standard, basic, and median Solar motion.

Based on the proper motion, Agekyan and Popovich (1993) presented a new method to determine the Solar apex. Ali and Sharaf (1998) used planetary nebulae as a stellar system to determine some Galactic constants like the Solar apex, speed of Solar motion, and the velocity ellipsoid parameters. Fehrenbach et al. (2001) have calculated Solar apex using the radial velocity data obtained with Fehrenbach Objective Prisms.

Elsanhoury et al. (2015) determined the velocity ellipsoid parameters of the Solar neighborhood white dwarfs, into which the Solar velocity and velocity dispersion are calculated for four subsamples, namely DA, non-DA, hot, and cool white dwarfs. On the other hand, Elsanhoury (2016) computed the velocity ellipsoid parameters for 790 late-type stars (cool stars) from CoRoT and 290 L dwarf stars (ultracool stars) from Sloan Digital Sky Survey (SDSS).

To determine the Solar motion parameters, we used a group of stellar associations with different spectral types (i.e. temp. scale). We used the method of velocity ellipsoid parameters VEPs (Elsanhoury et al., 2013) which is based on the space velocities ( $\mathrm{U}, \mathrm{V}, \mathrm{W}$ ) with respect to Galactic coordinates.

The structure of the paper is as follows: section 2 describe the data and the method of calculations, section 3 is devoted to the results and discussion. The conclusion is given in section 4.

## 2. DATA AND METHOD OF CALCULATIONS

### 2.1. DATA

We retrieved the data from the Extended Hipparcos Compilation (XHIP) crossreferences (Anderson and Francis, 2012) available in an electronic form at http: //vizier.u-strasbg.fr/viz-bin/VizieR-3 (V/137D). In this context, to get a net worksheet data which are necessary to build up our models for different spectral types, we have extracted stars with complete record of velocity vectors ( $\mathrm{U}, \mathrm{V}, \mathrm{W}$ ) $\mathrm{km} / \mathrm{s}$ and distances $\mathrm{d}(\mathrm{pc})$.

Therefore, we get a sample of 5100 main sequence stars which could be identified as follows:

1. Approximately $10 \mathrm{~B} 5\left(\mathrm{~T}_{\text {eff. }} \simeq 10500 \mathrm{~K}\right)$.
2. Approximately $349 \mathrm{~A} 0\left(\mathrm{~T}_{\text {eff. }} \simeq 9400 \mathrm{~K}\right)$.
3. Approximately $103 \mathrm{~A} 5\left(\mathrm{~T}_{\text {eff. }} \simeq 7800 \mathrm{~K}\right)$.
4. Approximately $286 \mathrm{~F} 0\left(\mathrm{~T}_{e f f .} \simeq 7300 \mathrm{~K}\right)$.
5. Approximately $780 \mathrm{~F} 5\left(\mathrm{~T}_{e f f .} \simeq 6500 \mathrm{~K}\right)$.
6. Approximately $747 \mathrm{G} 0\left(\mathrm{~T}_{\text {eff. }} \simeq 6000 \mathrm{~K}\right)$.
7. Approximately $1014 \mathrm{G} 5\left(\mathrm{~T}_{e f f .} \simeq 5700 \mathrm{~K}\right)$.
8. Approximately $1542 \mathrm{~K} 0\left(\mathrm{~T}_{e f f} \simeq 5250 \mathrm{~K}\right)$.
9. Approximately $269 \mathrm{~K} 5\left(\mathrm{~T}_{e f f} \simeq 4350 \mathrm{~K}\right)$.

### 2.2. METHOD OF CALCULATIONS

### 2.2.1. Velocity Ellipsoid Parameters

Based on the computational algorithm by Elsanhoury et al. (2013), we have computed the velocity ellipsoid parameters VEPs for the above sample of data as well as the Solar elements. A brief explanation of the algorithm will be given here. The average of the space velocities $(\bar{U}, \bar{V}, \bar{W})$ are given by

$$
\begin{equation*}
\bar{U}=\frac{1}{N} \sum_{i=1}^{N} U_{i} ; \bar{V}=\frac{1}{N} \sum_{i=1}^{N} V_{i} ; \bar{W}=\frac{1}{N} \sum_{i=1}^{N} W_{i} \tag{1}
\end{equation*}
$$

where N is the total number of stars in each spectral type. Let $\xi$ and its zero point coincide with the center of the distribution and let $l, m$ and $n$ be the direction cosines of the axis with respect to the shifted one, then the coordinates $Q_{i}$ of the point $i$, with respect to the $\xi$ axis, are given by

$$
\begin{equation*}
Q_{i}=l\left(U_{i}-\bar{U}\right)+m\left(V_{i}-\bar{V}\right)+n\left(W_{i}-\bar{W}\right) \tag{2}
\end{equation*}
$$

The generalization of the mean square deviation $\sigma^{2}$ is written as

$$
\begin{equation*}
\sigma^{2}=\frac{1}{N} \sum_{i=1}^{N} Q_{i}^{2} \tag{3}
\end{equation*}
$$

From the three Equations (1), (2), and (3) and after some manipulations we get

$$
\begin{equation*}
\sigma^{2}=\underline{x}^{T} B \underline{x} \tag{4}
\end{equation*}
$$

where $\underline{x}$ is the $(3 \times 1)$ direction cosines vector, and $B$ is $(3 \times 3)$ symmetric matrix elements $\mu_{i j}$ and

$$
\left.\begin{array}{ll}
\mu_{11}=\frac{1}{N} \sum_{i=1}^{N} U_{i}^{2}-(\bar{U})^{2} ; & \mu_{12}=\frac{1}{N} \sum_{i=1}^{N} U_{i} V_{i}-\overline{U V} ; \\
\mu_{13}=\frac{1}{N} \sum_{i=1}^{N} U_{i} W_{i}-\overline{U W} ; & \mu_{22}=\frac{1}{N} \sum_{i=1}^{N} V_{i}^{2}-(\bar{V})^{2} ;  \tag{5}\\
\mu_{23}=\frac{1}{N} \sum_{i=1}^{N} V_{i} W_{i}-\overline{V W} ; & \mu_{33}=\frac{1}{N} \sum_{i=1}^{N} W_{i}^{2}-(\bar{W})^{2} .
\end{array}\right\}
$$

Now, the necessary conditions for an extremum are

$$
\begin{equation*}
(B-\lambda I) \underline{x}=0 \tag{6}
\end{equation*}
$$

These are three homogenous equations in three unknowns have a nontrivial solution if and only if

$$
\begin{equation*}
D(\lambda)=|B-\lambda I|=0 \tag{7}
\end{equation*}
$$

Equation (7) is the characteristic equation for the matrix $B$, where $\lambda$ is the eigenvalue, $\underline{x}$ and $B$ are given by: $\underline{x}=\left[\begin{array}{l}l \\ m \\ n\end{array}\right]$ and $B=\left|\begin{array}{lll}\mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{12} & \mu_{22} & \mu_{23} \\ \mu_{13} & \mu_{23} & \mu_{33}\end{array}\right|$
Then the required roots (i.e. eigenvalues) are

$$
\left.\begin{array}{l}
\lambda_{1}=2 \rho^{\frac{1}{3}} \cos \left(\frac{\phi}{3}\right)-\frac{k_{1}}{3}  \tag{8}\\
\lambda_{2}=-\rho^{\frac{1}{3}}\left\{\cos \left(\frac{\phi}{3}\right)+\sqrt{3} \sin \left(\frac{\phi}{3}\right)\right\}-\frac{k_{1}}{3} \\
\lambda_{3}=-\rho^{\frac{1}{3}}\left\{\cos \left(\frac{\phi}{3}\right)-\sqrt{3} \sin \left(\frac{\phi}{3}\right)\right\}-\frac{k_{1}}{3}
\end{array}\right\}
$$

where

$$
\left.\begin{array}{l}
k_{1}=-\left(\mu_{11}+\mu_{22}+\mu_{33}\right), \\
k_{2}=\mu_{11} \mu_{22}+\mu_{11} \mu_{33}+\mu_{22} \mu_{33}-\left(\mu_{12}^{2}+\mu_{13}^{2}+\mu_{23}^{2}\right) \\
k_{3}=\mu_{12}^{2} \mu_{33}+\mu_{13}^{2} \mu_{22}+\mu_{23}^{2} \mu_{11}-\mu_{11} \mu_{22} \mu_{33}-2 \mu_{12} \mu_{13} \mu_{23} .
\end{array}\right\}
$$

and

$$
\begin{equation*}
\phi=\tan ^{-1}\left(\frac{\sqrt{x}}{r}\right) \tag{13}
\end{equation*}
$$

Depending on the matrix that control the eigenvalue problem [Equation (6)] for the velocity ellipsoid, we established an analytical expressions of some parameters in terms of the matrix elements.

### 2.2.2. Solar elements

1. Solar velocity: Consider $S_{\odot}$ is the Sun's velocity relative to the stellar associations, i.e.

$$
S_{\odot}=\sqrt{\bar{U}^{2}+\bar{V}^{2}+\bar{W}^{2}} \mathrm{~km} / \mathrm{s}
$$

2. Solar apex: Consider the Galactic longitude $l_{A}$ and Galactic latitude $b_{A}$ of the Solar apex are:

$$
\begin{aligned}
l_{A} & =\tan ^{-1}(-\bar{V} / \bar{U}) \\
b_{A} & =\sin ^{-1}\left(-\bar{W} / S_{\odot}\right)
\end{aligned}
$$

### 2.2.3. The velocity dispersion $\sigma_{j}, j=1,2,3$

The velocity dispersion is given by

$$
\begin{equation*}
\sigma_{j}=\sqrt{\lambda_{j}} \tag{14}
\end{equation*}
$$

### 2.2.4. The $\boldsymbol{l}_{j}, \boldsymbol{m}_{j}$, and $\boldsymbol{n}_{j}$ parameters

The direction cosines $l_{j}, m_{j}$, and $n_{j}$ are given by

$$
\begin{gather*}
l_{j}=\left[\mu_{22} \mu_{33}-\sigma_{j}^{2}\left(\mu_{22}+\mu_{33}-\sigma_{j}^{2}\right)-\mu_{23}^{2}\right] / D_{j} ; j=1,2,3  \tag{15}\\
m_{j}=\left[\mu_{23} \mu_{13}-\mu_{12} \mu_{33}+\sigma_{j}^{2} \mu_{12}\right] / D_{j} ; j=1,2,3  \tag{16}\\
n_{i}=\left[\mu_{12} \mu_{23}-\mu_{13} \mu_{22}+\sigma_{i}^{2} \mu_{13}\right] / D_{i} ; j=1,2,3 \tag{17}
\end{gather*}
$$

where

$$
\begin{aligned}
& \quad D_{j}^{2}=\left(\mu_{22} \mu_{33}-\mu_{23}^{2}\right)^{2}+\left(\mu_{23} \mu_{13}-\mu_{12} \mu_{33}\right)^{2}+\left(\mu_{12} \mu_{23}-\mu_{13} \mu_{22}\right)^{2} \\
& +2\left[\left(\mu_{22}+\mu_{33}\right)\left(\mu_{23}^{2}-\mu_{22} \mu_{33}\right)+\mu_{12}\left(\mu_{23} \mu_{13}-\mu_{12} \mu_{33}\right)+\mu_{13}\left(\mu_{12} \mu_{23}-\mu_{13} \mu_{22}\right] \sigma_{j}^{2}\right. \\
& \quad+\left(\mu_{33}^{2}+4 \mu_{22} \mu_{33}+\mu_{22}^{2}-2 \mu_{23}^{2}+\mu_{12}^{2}+\mu_{13}^{2}\right) \sigma_{j}^{4}-2\left(\mu_{22}+\mu_{33}\right) \sigma_{j}^{6}+\sigma_{j}^{8}
\end{aligned}
$$

### 2.2.5. Projected distance

The distance from the Galactic center $\mathrm{R}_{g c}$ and the projected distances on the Galactic plane from the $\operatorname{Sun}\left(\mathrm{X}_{\odot}, \mathrm{Y}_{\odot}\right)$ and the distance from the Galactic plane $\left(\mathrm{Z}_{\odot}\right)$ could be determined from the following (Mihalas and Binney, 1981)

$$
\begin{aligned}
& X_{\odot}=R_{\odot} \cos (b) \cos (l) \\
& Y_{\odot}=R_{\odot} \cos (b) \sin (l) \\
& Z_{\odot}=R_{\odot} \sin (b) \\
& R_{g c}^{2}=R_{o}^{2}+R_{\odot}^{2}-2 R_{o} R_{\odot} \cos (l)
\end{aligned}
$$

where $\mathrm{R}_{o}$ is the Sun's distance from the Galactic center ( 8.5 kpc ).

## 3. RESULTS AND DISCUSSION

Based on the method described in section 2 we elaborated MATHEMATICA code to compute the parameters of the Solar motion and kinematical properties. Figure 1, shows our calculated Solar velocities versus different spectral types with error bars (i.e. standard error) of the Solar velocity. We listed the results in Tables 1 and 2 , where row 1 is the total number of stars in each class, row 2 is the effective temperature of stars, row 3 is the average distance for each group (pc), rows 4, 5, and 6 are the average space velocities due to Galactic coordinates, rows $7,8,9,10$, and 11
are the elements of Solar motion, rows $12,13,14,15$ and 16 are the values of VEPs for different spectral classes, rows 17,18 , and 19 are the projected distances ( pc ) and finally row 20 is the distance to the Galactic center (pc).

Table 3, presents our ratio of velocity dispersion $\left(\sigma_{2} / \sigma_{1}\right)$ for each spectral type, which related to the important quantities in stellar kinematics, i.e. Oort's constants through $\left(\sigma_{2} / \sigma_{1}\right)^{2}=-B /(A-B)$. For the sake of comparison, we listed in Table 4 the values of A and B constants according to Olling and Merrifield (1998) and we see that, the dispersion ratios $\left(\sigma_{2} / \sigma_{1}\right)$ have values in the range 0.65 to 0.74 , which are in good agreement with our calculations for different spectral types except for B5 and A5, which may be attributed to the small number of stars (10 and 103 points, respectively).

Many features could be obtained from the results listed in Table 1 and 2. For all spectral types the velocity dispersions $\left(\sigma_{1}, \sigma_{1}, \sigma_{3}\right)$ obey the inequalities $\sigma_{1}>\sigma_{2}>$ $\sigma_{3}$, and as remarked, the stars earlier than F5 have smaller velocity dispersion than the later one. This could be understood from the fact that early and late type stars have different evolution scenarios.

The longitude of the vertex of the velocity ellipsoid $\left(l_{A}^{o}\right)$ calculated for our sample indicates that the principal axis points in the direction of the Galactic center for late-type stars. For early type stars, they show a large departure from the Galactic center.


Fig. 1 - Solar velocity versus different spectral types.

Table 1
Solar motions, VEPs, and projected distances for different early spectral classes

| Parameters | B5 | A0 | A5 | F0 | F5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| N | 10 | 349 | 103 | 286 | 780 |
| $\mathrm{T}_{\text {eff. }}\left(\mathrm{K}^{o}\right)$ | 10500 | 9400 | 7800 | 7300 | 6500 |
| $R_{\odot}(p c)$ | 279.758 | 205.02 | 150.043 | 131.244 | 104.259 |
| $\bar{U}_{k m / s}$ | $\begin{array}{ll} -1.75 & \pm \\ 0.09 & \end{array}$ | $\begin{array}{ll} \hline-5.87 & \pm \\ 0.30 & \end{array}$ | $\begin{array}{ll} \hline-14.4 & \pm \\ 0.72 & \end{array}$ | $\begin{array}{ll} \hline-11.91 & \pm \\ 0.60 & \end{array}$ | $\begin{array}{ll} \hline-8.73 & \pm \\ 0.44 & \end{array}$ |
| $\overline{\bar{V}}_{k m / s}$ | $\begin{array}{ll} \hline-23.37 & \pm \\ 1.17 & \end{array}$ | $\begin{array}{ll} \hline-12.21 & \pm \\ 0.62 & \end{array}$ | $\begin{array}{ll} -12.92 & \pm \\ 0.65 & \end{array}$ | $\begin{array}{ll} \hline-11.48 & \pm \\ 0.58 & \end{array}$ | $\begin{array}{ll} \hline-14.58 & \pm \\ 0.73 & \end{array}$ |
| $\bar{W}_{k m / s}$ | $-9.1 \pm 0.46$ | $\begin{array}{ll} \hline-7.05 & \pm \\ 0.35 & \\ \hline \end{array}$ | $\begin{array}{ll} \hline-6.62 & \pm \\ 0.33 & \\ \hline \end{array}$ | $\begin{array}{ll} -7.03 & \pm \\ 0.35 & \\ \hline \end{array}$ | $\begin{array}{ll} \hline-7.75 & \pm \\ 0.39 & \\ \hline \end{array}$ |
| $S_{\odot}(\mathrm{km} / \mathrm{s})$ | $\begin{array}{ll} \hline 25.14 & \pm \\ 2.51 & \end{array}$ | $\begin{array}{ll} \hline 15.27 & \pm \\ 1.53 & \end{array}$ | $\begin{array}{ll} 20.44 & \pm \\ 2.04 & \end{array}$ | $\begin{array}{ll} \hline 17.97 & \pm \\ 1.80 & \\ \hline \end{array}$ | $\begin{array}{ll} \hline 18.68 & \pm \\ 1.87 & \\ \hline \end{array}$ |
| $l_{A}^{o}$ | 85.73 | 64.31 | -41.91 | 43.96 | 59.12 |
| $b_{A}^{o}$ | 21.23 | 27.51 | 18.90 | 23.01 | 24.51 |
| $\alpha_{A}^{o}$ | 284.51 | 268.03 | 269.56 | 266.36 | 270.02 |
| $\delta^{\circ}{ }_{A}$ | 55.68 | 38.44 | 16.21 | 19.57 | 33.13 |
| $\left(\lambda_{1}, \lambda_{2}, \lambda_{3}\right)_{k m / s}$ | $\begin{aligned} & 2874.44, \\ & 141.09, \\ & 25.5 \end{aligned}$ | $\begin{aligned} & 446.34 \\ & 204.81 \\ & 67.15 \end{aligned}$ | $\begin{aligned} & \text { 2561.88, } \\ & 220.13, \\ & 140.57 \end{aligned}$ | $\begin{aligned} & 825.3 \\ & 361.35 \\ & 296.27 \end{aligned}$ | $\begin{aligned} & 1176.8, \\ & 526.152, \\ & 237.71 \end{aligned}$ |
| $\left(\sigma_{1}, \sigma_{2}, \sigma_{3}\right)_{k m / s}$ | $\begin{aligned} & \hline 53.61, \\ & 11.88,5.05 \end{aligned}$ | $\begin{aligned} & \text { 21.13, } \\ & 14.31,8.20 \end{aligned}$ | $\begin{aligned} & \hline 50.62, \\ & 14.84, \\ & 11.86 \end{aligned}$ | $\begin{aligned} & 28.73, \\ & 19.01, \\ & 17.22 \end{aligned}$ | $\begin{aligned} & \hline 34.305 \\ & 22.94, \\ & 15.42 \end{aligned}$ |
| $\left(l_{1}, m_{1}, n_{1}\right)_{\text {deg }}$ | $\begin{aligned} & 0.11,-0.93, \\ & -0.35 \end{aligned}$ | $\begin{array}{ll} \hline 0.66, & 0.71, \\ 0.26 \end{array}$ | $\begin{aligned} & 0.82,0.57,- \\ & 0.11 \end{aligned}$ | $\begin{array}{ll} 0.91, & 0.31, \\ 0.27 & \end{array}$ | $\begin{array}{ll} \hline 0.92, \quad 0.39, \\ 0.04 \end{array}$ |
| $\left(l_{2}, m_{2}, n_{2}\right)_{\text {degs }}$ | $\begin{aligned} & \hline-0.83, \\ & -0.28,0.48 \end{aligned}$ | $\begin{aligned} & -0.75,0.60, \\ & 0.30 \end{aligned}$ | $\begin{aligned} & -0.41,0.70, \\ & 0.60 \end{aligned}$ | $\begin{aligned} & -0.05, \\ & -0.60,0.81 \end{aligned}$ | $\begin{aligned} & -0.39,0.87, \\ & 0.29 \end{aligned}$ |
| $\left(l_{3}, m_{3}, n_{3}\right)_{\text {deg }}$ | $\begin{aligned} & 0.55,-0.24, \\ & 0.80 \end{aligned}$ | $\begin{aligned} & \text { 0.06, -0.39, } \\ & 0.92 \end{aligned}$ | $\begin{aligned} & 0.41,-0.44, \\ & 0.81 \end{aligned}$ | $\begin{aligned} & \hline 0.41,-0.75, \\ & -0.52 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.08,-0.29, \\ & 0.95 \end{aligned}$ |
| $X_{\odot}(p c)$ | $\begin{array}{ll} \hline 26.25 & \pm \\ 5.12 & \end{array}$ | $4.11 \pm 2.03$ | $6.27 \pm 2.50$ | $3.94 \pm 1.98$ | $2.25 \pm 1.5$ |
| $Y_{\odot}(p c)$ | $\begin{array}{ll} -10.92 & \pm \\ 3.30 & \end{array}$ | $\begin{array}{ll} 18.91 & \pm \\ 4.35 & \end{array}$ | $\begin{array}{ll} \hline-3.52 & \pm \\ 1.88 & \end{array}$ | $\begin{array}{ll} \hline-4.62 & \pm \\ 2.15 & \end{array}$ | $\begin{array}{cc} -0.86 & \pm \\ 0.009 & \end{array}$ |
| $Z_{\odot}(p c)$ | $\begin{array}{ll} \hline 33.07 & \pm \\ 5.75 & \end{array}$ | $\begin{array}{ll} \hline-0.017 & \pm \\ 0.001 & \end{array}$ | $\begin{array}{ll} \hline-8.25 & \pm \\ 2.87 & \end{array}$ | $4.39 \pm 2.10$ | $\begin{array}{cc} \hline-0.60 & \pm \\ 0.008 & \end{array}$ |
| $R_{g c}(k p c)$ | 8.484 | 8.515 | 8.484 | 8.502 | 8.498 |

Table 2
Solar motions, VEPs, and projected distances for different late spectral classes

| Parameters | G0 | $\mathbf{G 5}$ | $\mathbf{K}$ | $\mathbf{K 5}$ |
| :--- | :--- | :--- | :--- | :--- |
| N | 747 | 1014 | 1542 | 269 |
| $\mathrm{~T}_{e f f .}\left(\mathrm{K}^{o}\right)$ | 6000 | 5700 | 5250 | 4350 |
| $R_{\odot}(p c)$ | 86.90 | 107.579 | 180.205 | 199.087 |
| $\bar{U}_{k m / s}$ | $-10.76 \pm 0.54$ | $-8.64 \pm 0.43$ | $-7.53 \pm 0.38$ | $-5.63 \pm 0.28$ |
| $\bar{V}_{k m / s}$ | $-20.98 \pm 1.05$ | $-22.15 \pm 1.11$ | $-20.24 \pm 1.01$ | $-26.54 \pm 1.33$ |
| $\bar{W}_{k m / s}$ | $-7.17 \pm 0.36$ | $-6.82 \pm 0.34$ | $7.13 \pm 0.36$ | $-8.66 \pm 0.43$ |
| $S_{\odot}(k m / s)$ | $24.64 \pm 2.46$ | $24.73 \pm 2.47$ | $22.74 \pm 2.27$ | $28.48 \pm 2.85$ |
| $l_{A}^{o}$ | 62.84 | 68.69 | 69.61 | 78.03 |
| $b_{A}^{o}$ | 16.91 | 16.00 | 18.28 | 17.71 |
| $\alpha_{A}^{o}$ | 280.01 | 283.66 | 281.32 | 286.01 |
| $\delta_{A}^{o}$ | 33.86 | 38.73 | 40.38 | 47.67 |
| $\left(\lambda_{1}, \lambda_{2}, \lambda_{3}\right)_{k m / s} s$ | 1766.23, | 1638.19, | 1437.21, | 1848.78, |
|  | $854.49,565.23$ | 1192.13, | $857.08,342.04$ | 1398.28, |
| $\left(\sigma_{1}, \sigma_{2}, \sigma_{3}\right)_{k m / s} / 42.03, \quad 29.23$, | $40.48, \quad 34.53$, | $37.91, \quad 29.28$, | 43.00, |  |
|  | 23.78 | 19.71 | 18.49 | 22.27 |
| $\left(l_{1}, m_{1}, n_{1}\right)_{\mathrm{deg}}$ | $0.84,0.54,0.06$ | $0.85,0.52,0.07$ | $0.84,0.53,0.11$ | $0.65,0.76,0.08$ |
|  |  |  |  | 496.16 |
| $\left(l_{2}, m_{2}, n_{2}\right)_{\mathrm{deg}}$ | -0.53, | 0.80, | -0.52, | 0.83, |
|  | 0.28 | -0.54, | 0.81, | -0.76, |
| $\left(l_{3}, m_{3}, n_{3}\right)_{\operatorname{deg}}$ | 0.10, | -0.27, | 0.05, | 0.24 |
| $X_{\odot}(p c)$ | 0.96 | $-0.69 \pm 0.008$ | $3.18 \pm 1.78$ | 0.04, |
| $Y_{\odot}(p c)$ | $0.13 \pm 0.003$ | $-1.80 \pm 1.34$ | -0.27, | 0.02, |
| $Z_{\odot}(p c)$ | $-0.98 \pm 0.009$ | $-7.12 \pm 2.70$ | $0.57 \pm 0.007$ | -0.13, |
| $R_{g c}(k p c)$ | 8.501 | 8.507 | 8.501 | 0.99 |

Table 3
Velocity dispersions for different spectral types

| Spectral Types | $\mathbf{S}_{\odot}$ | $\left(\sigma_{2} / \sigma_{1}\right)$ |
| :---: | :---: | :---: |
| B5 | $25.14 \pm 2.51$ | 0.22 |
| A0 | $15.27 \pm 1.53$ | 0.68 |
| A5 | $20.44 \pm 2.04$ | 0.30 |
| F0 | $17.97 \pm 1.80$ | 0.66 |
| F5 | $18.68 \pm 1.87$ | 0.67 |
| G0 | $24.64 \pm 2.46$ | 0.70 |
| G5 | $24.73 \pm 2.47$ | 0.85 |
| K0 | $22.74 \pm 2.27$ | 0.77 |
| K5 | $28.48 \pm 2.85$ | 0.87 |

Table 4
Oort's constants, Olling and Merrifield (1998)

| $A\left(\mathrm{~km} \mathrm{~s}^{-1} \mathrm{kpc}^{-1}\right)$ | $B\left(\mathrm{~km} \mathrm{~s}^{-1} \mathrm{kpc}^{-1}\right)$ | $\sigma_{2} / \sigma_{1}$ |
| :---: | :---: | :---: |
| 14.5 | -12 | 0.65 |
| 12.6 | -13.2 | 0.71 |
| 14.8 | -12.4 | 0.67 |

## 4. CONCLUSION

For a sample of 1528 early-type stars and 3572 late-type stars, we have calculated the Solar elements, VEPs, and projected distances to the Galactic plane. From the results, we may say that all of these parameters are temperature dependence. The results reached could be drawn through the following points:

1. As appeared from Fig. 1, the Solar velocity is increased as temperature decreased (from early to late-type stars).
2. For each model, we have computed the velocity dispersion $\sigma_{j} ; j=1,2$, and 3 , based on the matrix elements $\lambda_{j} ; j=1,2$, and 3 .. So, the ratio $\left(\sigma_{2} / \sigma_{1}\right)$ is computed for each class. Our results are in good agreement with the values calculated by different authors, except for classes B5 and A5, which may be attributed to the small sample of data (10 and 103).
3. We have estimated the average distance $\mathrm{R}_{\odot}$ for each set, therefore the projected distances $\left(\mathrm{X}_{\odot}, \mathrm{Y}_{\odot}, \mathrm{Z}_{\odot}\right)$ to the Galactic plane and the distances to the Galactic center $\mathrm{R}_{g c}$ have been also computed.

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