FOURIER DECOMPOSITION AND PERIOD CHANGE FOR SIX LOW AMPLITUDE CEPHEIDS

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Abstract. Fourier decomposition Technique is used for light curves to obtain rigorous estimates of the pulsation amplitudes of some Cepheids. The pulsating stars, T Vul, U Aql, η Aql, ζ Gem, X Lac, and T Mon, are now known to undergo slow changes in their amplitude. We have used all observations available from the different international Databases, to test the hypothesis that an increase/decrease in amplitude would result in an increase/decrease in period, as a result of non-linear effects in the pulsation. The amplitude stability or amplitude variation, in addition to other processes such as rates of period change of the six Cepheids are demonstrated to be in general agreement with predictions from stellar evolutionary models. Also, common features of such pulsating stars, *i.e.* random errors in measuring times of light maxima which fluctuated in period, are studied.

Key words: variable stars – Cepheids variable – stellar evolution.

1. INTRODUCTION

In the last decade, secular decrease in the pulsational amplitude was revealed for two classical Cepheids: Polaris and Y Oph. Secular decrease in the pulsational amplitude of Polaris was discussed by Ferro (1983), the star's visual light amplitude had diminished from about 0.1 magnitude to about 0.05 magnitude in 50 years or more. Fernie *et al.* (1993) predicted that Polaris will complete cessation of pulsation in the mid-nineties and confirmed their prediction by subsequent observations, but in 1995, Polaris still had detectable radial velocity amplitude. Kamper and Fernie (1998) discovered a serious error in their earlier paper. They corrected this error and confirmed that no longer predict cessation of pulsation in 1995. A comprehensive study of V473 Lyr shows that the amplitude of this Cepheid varies periodically on a time scale of about 1400 days (Burki *et al.* , 1986). All these studies motivated us to focus on the low amplitude behavior of the sample of Cepheids under study. The primary purpose of this paper is to study the pulsational amplitude stability and to utilize the Fourier fits of light curves for the six Cepheids: U Aql, η Aql, ζ Gem, X Lac, T Mon, and T Vul. These stars have long history of observations (at least 100 year observations). The secondary purpose is to study the behavior of their period. In all cases the evolution of stars through the Cepheid instability strip should be associated with gradual changes. In the overall dimensions and hence periods of pulsation. The period increases for evolution toward the cool edge of the instability strip as the stellar radius grows, and decreases for evolution toward the hot edge as the stellar radius decreases. The O-C (plots of the differences between observed times of light maximum and those computed from a linear ephemeris) or Eddington–Plakidis method, discussed below, is used to calculate the period changes, by phase shifts of each cycle. The phase shift technique depends on the phased observations over a restricted number of cycles. It is used to construct light curve for the variables that were then matched to a standard light curve to detect phase shifts indicative of period changes which named Hertzsprung's method. Also Turner *et al.* (1999) used a set of high quality observations as a standard and phase space by using two methods, the first is the least square technique and the second by determining the time of minima or maxima shift. In more details the calculations of the O-C values are described below. Our paper discussed the following topics. The first one is the amplitude stability and the second one studied period change. In section 2, we discussed the Fourier technique which is used to calculate the amplitudes of U Aql, η Aql, ζ Gem, X Lac, T Mon, and T Vul . The amplitude behaviors of Cepheids variable for V and B are represented in Fig.1 and Table 2. In Section 3 we described the period change for the six stars, random cycle to cycle period fluctuations in addition to the O-C light curves and residual for O-C are also studied.

2. FOURIER TECHNIQUE

Fourier decomposition technique has been widely used for study of the light curves of pulsating stars, for a review see Simon (1988). This technique can provide quantitative parameters to define the shape of light curves. It is for this reason a powerful tool for the classification purposes. Fourier decomposition technique represented by eq. (1) was applied to the collected light curves of studied stars in B and V bands.

$$m(t) = m_0 \sum_{i=1}^{\infty} A_i \sin(2\pi(f_i(t) + \phi_i)), \quad (1)$$

where $m(t)$ is the magnitude observed at time *t*, m_o the mean magnitude, A_i is the amplitude of the its component (harmonic), f is the frequency ($f = 1/P$, where P is the period of light variation), and ϕ_i is the phase of the i^{th} component. Fourier parameters can be subdivided into two groups: the amplitude ratios $R_{ij} = A_i/A_j$ and the phase differences $\phi_{ij} = i\phi_j - j\phi_i$. We calculated the Fourier coefficient because

Table 1

Basic information about studied stars and Archival observation data sets used in this paper

Star Name	Log P (day)	Type	Min Mag.	Max Mag.	Start Obs.	End Obs.
U Aql	0.846582	DECP	6.860	6.08	09-1897	10-2007
η Aql	0.855927	DECP	4.390	3.48	11-1784	08-2013
ζ Gem	1.006472	DECP	4.018	3.62	03-1845	09-2008
X Lac	0.735997	DECPS	8.650	8.20	12-1906	09-2008
T Mon	1.431760	DECP	6.620	5.58	08-1872	12-2007
T VUL	0.646939	DECP	6.090	5.41	12-1885	08-2014

the larger value of R_{21} indicates stronger deviations from a single sinusoid and thus indicates higher degree of asymmetry in the light curve. The Fourier parameters calculated for the light curves of the stars are tabulated in Table 2.

Two Fortran programs in addition to *Period04* software are used to determine the Fourier parameters. All were used to determine the numerical parameters describing the light curves of the studied stars. The results of the sample of Cepheids are represented in Table 2. In our reduction we bear in mind that the data should cover the light curves minimum and maximum parts to enable reliable amplitude estimation using the Fourier decomposition technique. Moreover both B and V (and/or visual) were used to check any claimed amplitude and period change. Amplitudes of the individual Fourier-harmonic, phase differences, amplitude ratio, were determined in the B and V photometric bands (or nearest band to them). So, Cepheids' amplitude display a period dependence as well as a dependence on location within the strip, a natural consequence of an effect tied to both surface gravity and pulsation efficiency.

2.1. FOURIER RESULTS

The long term behaviors for the six Cepheid stars are listed in Table 1. By using all available published observations for these stars. We calculated the long term behavior of the harmonic amplitude for these stars and the results are shown in fig 1. When we bounded our study, we cant be able to adopt any fixed secular variation in amplitude except little secular variation but we attributed this variation to the heterogeneous observations and some scatter in observations in addition to the gap between observations collected for the studied stars. in all that we can't see clear variation in amplitude in our samples. As shown in Fig.1, the mathematical linear relation in V color for the six stars tabulated in Table 2. The gradient or slope for X Lac and T Mon Cepheids are (0.030±0.013) and (-0.03±0.01) and its intersect are (-6.9±3.3), (7.7±2.4),respectively. These slopes are higher than others, but the lowest gradient or slope and intersect in U Aql are (-0.004 ±00.423) and it is (1.317±1.123). In spite of the largest observation for these stars for more than hundred year we can't see any clear changes, this means that stellar amplitude for the six stars are at least stable or/and we needful more observations in the future to see any little amplitude variations. But a selection of Fourier series were matched to the data differing only in the number of terms used in the series fit. The best fitting solution is for a series containing terms up to third order. Given the nature of observational scatter, it is not clear that the adoption of higher order terms in the Fourier series fit has any underlying significance as noted by Simon and Lee (1981). The derived Fourier parameters and the linear fitting are presented in Table 2. We use all data available from literature and the new catalogues (OGLE, ASAS, Hipparcos, and AAVSO) for amplitude variability to study the chosen star sample and the result is negative to see any clear variation in the amplitude except secular variations in some stars as already tabulated in Table 2.

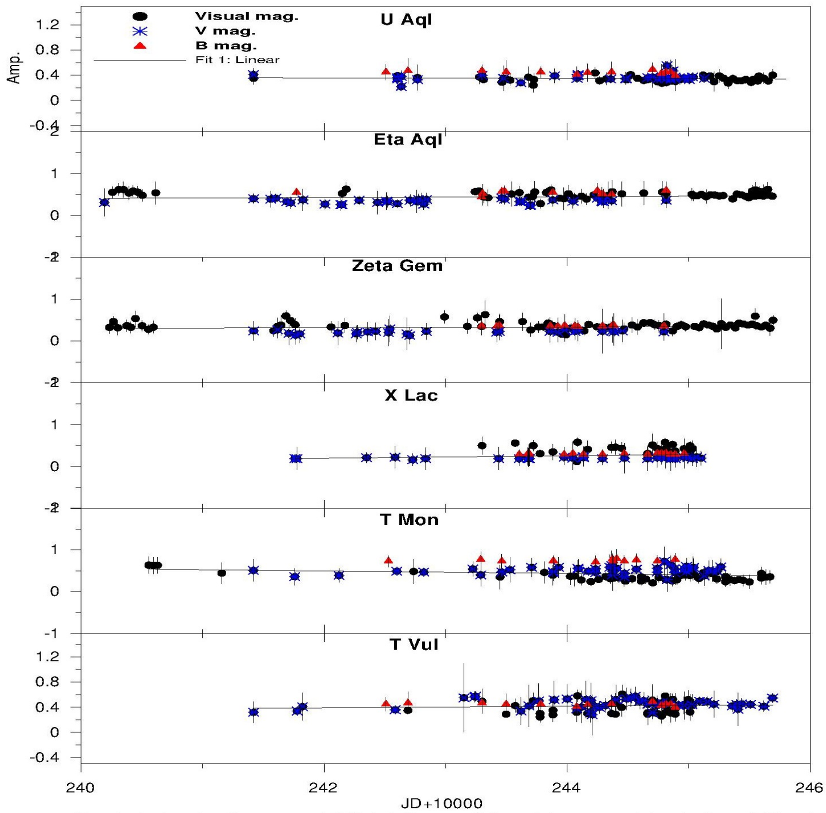

Fig. 1 – The stellar behavior of amplitude for the stars U Aql, η Aql, ζ Gem, X Lac, T Mon, and T Vul in V and B Jonson-Morgan systems.

Table 2

Mean Fourier parameters for studied stars in V and B bands and Mathematical linear fit

Star	V_V	R_{21}(V)	R_{21}(B)	R_{31}(B)	R_{31}(V)	ϕ_{21}(B)	ϕ_{21}(V)	$Ax10^{-3}$ ± Err.	B
U Aql	0.36	0.322	0.449	0.227	0.191	4.82	4.76	-03.961	1.317
								00.423	1.123
η Aql	0.34	0.360	0.354	0.199	0.141	4.89	4.73	10.231	-2.0479
								04.151	1.497
ζ Gem	0.22	0.097	0.062	0.079	0.046	1.04	1.06	08.032	-1.626
								06.67	1.624
X Lac	0.20	0.229	0.199	0.111	0.078	4.44	4.242	9.523	-6.944
								13.511	3.297
T Mon	0.47	0.371	0.363	0.225	0.185	4.46	4.22	-29.580	7.654
								09.725	2.391
T Vul	0.32	0.353	0.338	0.146	0.135	4.26	4.171	2.255	-2.574
								01.259	3.0783

3. PERIOD VARIATIONS

The first report on the discover of Cepheids' secular pulsation period change was made by Hertzsprung (1919) who analyzed photometric observations of the variable star Delta Cephei in the years 1785 to 1911. After that Cepheids period changes becomes an important tool for the characterization of individual members of the class. Both mass and strip crossing mode of Cepheid dictate the rate and direction of its period changes, so the rate of period change for individual Cepheids is of large value for establishing both evolutionary status and position in the instability strip, as well as for testing evolutionary models (see Turner (1998)). The observed parabolic trends in Cepheids' (O-C) diagram have been recognized for the past half-century as evidence of the evolution of such stars through the instability strip (Parenago, 1958; Struve, 1959; Erleksova and Irkaev, 1982).

3.1. PERIOD CHANGE

Period change is very important because at least, observations of period changes in Cepheids have been matched with some confidence to the evolutionary models of massive stars in various crossings of the instability strip (*e.g.* Turner (1998); Turner and Berdnikov (2001) and Turner and Berdnikov (2004)) this is important to identify the direction of strip crossing for individual variables. The data bank on Cepheids photometry permit us to study period variations. The sample we used for period change studies contains six Cepheids with observations covering a period between 100 to 230 years (see Table 1). In the present work we studied the period behavior for our sample of low amplitude Cepheids. We used all time of maximum published by Szabados (1977), Szabados (1980), Szabados (1981), Szabados (1991), Turner *et al.* (2006), Turner and Berdnikov (2001), Turner and Berdnikov (2004) and Abdel-Sabour (2005), in addition to observations from AVVSO, ASAS, and OGEL organization. We tried to check the period behavior and also updated the linear and quadratic (polynomial) ephemeris for each of the star under focus. The O-C residuals for all studied stars are calculated using linear ephemeris Szabados (1977), Szabados (1980), Szabados (1981), Szabados (1991) which written on its figures. The scatters in the O-C diagram for some stars is duo to the binary system or as a result of simple observational scatter, or they might be evidence of random cycle-to-cycle fluctuations in the periodicity of the light variation. O-C data are closely approximated by a parabolic trend which reflecting a regular period decrease, or period increase. New parabolic elements of these Cepheids' are given in Tables 4 according to the following relation:

$$HJD_{max} = M_o + PE + QE^2 \tag{2}$$

where M_0 is a new epoch, P is the new period and Q is a term that can be used to measure the values of period change (dP/dt) in seconds per year, and given by, $dP/dt = (2Q/P) \times 365.25 \times 24 \times 60 \times 60$. The quadratic dependence of the pulsation period on the evolutionary time ephemeris (E) implies the linear change of P(dot) which decreases during the first crossing of the instability strip and increases during the next crossing. Table 4 included the new linear and quadratic (polynomial) ephemeris for all studied stars. We can summarize our results as: star name, linear and polynomial ephemeris M_0, P, Q , log period, and the limited observations.

Fig. 2 shows the O-C diagrams for the stars, U Aql, η Aql, ζ Gem, X Lac, T Mon, and T Vul (from top to bottom, respectively), including observations compiled by literature and data sources (AAVSO and ASAS) and the parabola in each case represents a regression fit.

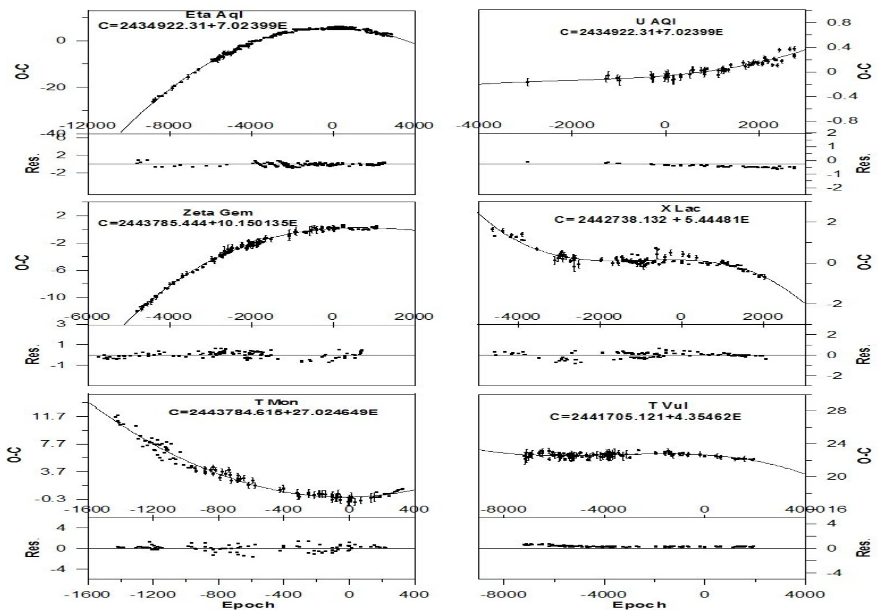

Fig. 2 – The O-C diagrams for the stars, U Aql,η Aql,ζ Zeta Gem, X Lac, T Mon, and T Vul (from top to bottom), including observations compiled by literature and data sources (AAVSO and ASAS) and the parabola in each case represents a regression fit.

3.2. RANDOM CYCLE-TO-CYCLE PERIOD FLUCTUATIONS

Random cycle-to-cycle period fluctuations were determined from the time minimum/or maximum using the method given by Edington and Plakidis (1929). In that method one examines temporal differences $a(r)$ of each of the r^{th} observed light maxima from the established trend and computes, without regard to sign, the accumulated delays ($U(x)$) which is represented by equation (3).

$$U(x) = a(r+x) - a(r), \tag{3}$$

This is the relation between maxima separated by x cycles. The mean value for all accumulated delays between light maxima separated by x cycle ,$\prec U(x) \succ$, should be correlated with any random fluctuations in period, according to Eddington and Plakidis, such that $\prec \cdot U(x) \cdot \succ^2 = 2a^2 + xe^2$. Where *a* represents the magnitude of random errors in measuring the times of light maxima. This technique has been applied for huge number of stars for examples T and δ Cep, and SV Vul by Turner and Berdnikov (2001), Turner and Berdnikov (2002), Turner *et al.* (2006), and Mira variables, RV Tau and RU Cam by Percy and Jonathan (1998). It appears that random fluctuations in period is common feature of such pulsating stars. The resulting diagram for U Aql, η Aql, ζ Gem, X Lac, T Mon, and T Vul, using the same data represented in Fig. 2 , is shown in Fig. 3.

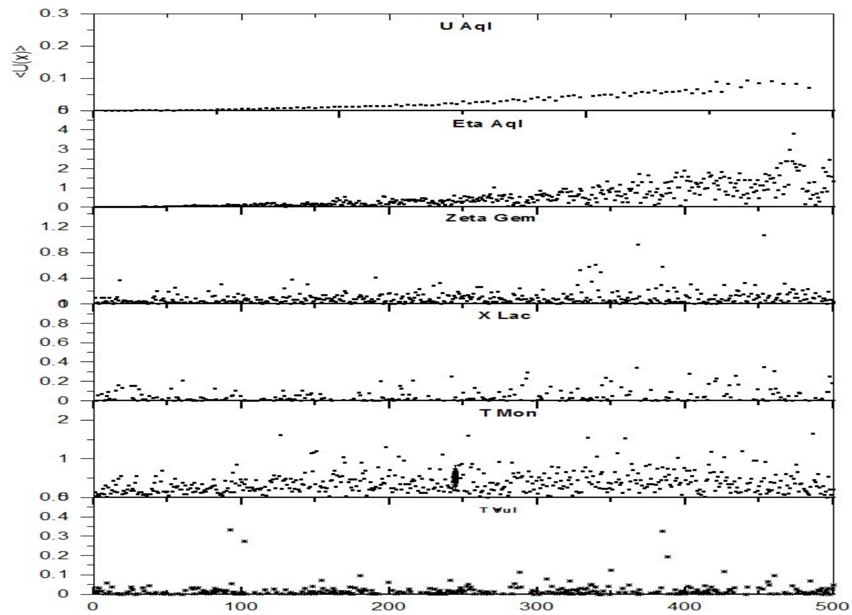

Fig. 3 – $\prec$ U(x) $\succ^2$ diagram for U Aql, η Aql, ζ Gem, X Lac, T Mon, and T Vul, based on O–C data and the linear beast fit.

4. CONCLUSION AND DISCUSSION

We have identified a sample of six old population I fundamental mode pulsators (s-Cepheids). These are U Aql, η Aql, ζ Gem, X Lac, T Mon, and T Vul.

Table 3

The least square fit for equation $HJD_{max} = M_o + PE + QE^2$ to the data results in an improved ephemeris for Cepheids under study , Log P(dot) and fit straight line for cycle-to-cycle fluctuation

Star Name	Quadratic (Polynomial) elementLog	log P(dot)
U Aql	C = 2434922.2030+7.02407 E +1.941 x 10^{-8}E^2	-0.7479±(0.0185)
η Aql	C = 2442794.9986+7.176844 E + 3.323 x 10^{-8} E^2	-1.1442±(0.1280)
ζ Gem	C =2443785.5586+10.150238 E 5.034x10^{-7} E^2	0.4559±(0.0377)
X Lac	C = 2442738.2289+ 5.44498 E 7.697 x 10^{-8} E^2	0.6622±(0.0815)
T Mon	C =2443784.6954+27.025374 E +5.894 x 10^{-6} E^2	1.1555±(0.3629)
T Vul	C = 2443090.0923+4.435471 E - 2.962 x 10^{-8} E^2	-0.7898±(0.0135)

We have examined the possibility of using Fourier Parameters to test the amplitude variation for these stars. As known, the luminosity amplitude has been recognized as a key parameter for assessing the pulsation properties of classical Cepheids. The first study of Cepheid amplitudes as a function of position in the strip was done by Kraft (1963). He concluded that small-amplitude Cepheids were found only on the hot edge of the strip. This means the amplitude change depends on the location in the instability strip. So we can use a period luminosity amplitude relation to estimate classical Cepheids distances. All subsequent amplitude maps of the instability strip by Hofmeister (1967), Sandage and Tammann (1971), Payne-Gaposchkin (1974), Pel and Lub (1978), Turner (2001), and Sandage *et al.* (2004) have produced similar results, namely, a sharp rise to maximum amplitude on the hot edge of the strip followed by a more gradual decline toward the cool edge. On the other hand, Fernie (1990) investigated a large sample of Galactic Cepheids suggesting that for a given period the largest amplitudes are possibly attained close to the centre of the instability strip. But tell now, Fernie suggestion is not tight. Our samples are in the fundamental pulsation mode. So, they have an increasing amplitude when crossing the instability strip from blue to red edge and decreasing amplitude when crossing from red to blue. We have found that, the long term behavior of the studied stars show predict that, the stars, η Aql, ζ Gem, X Lac, and T Vul, show a secular increase in their harmonic Fourier amplitude, while U Aql, and T Mon show a secular decrease.

From the results of the period behavior we updated the linear and quadratic (polynomial) ephemeris for all studied stars which can use in future to calculate the residuals for any observations. The stars U Aql, X Lac, and T Mon show a secular increase in their period with the rates of 1.941×10^{-8} day/cycle, 2.519×10^{-8} day/cycle, and 1.941×10^{-8} day/cycle, respectively, while the stars η Aql,ζ Gem, and T Vul show a secular decrease with the rates of 6.452×10^{-10} day/cycle, 6.131×10^{-9}, and 2.962×10^{-8} day/cycle, respectively. The period variation of Cepheids variable is very important to understand its evolutionary state. It needs further long term photometric monitoring for accurate epochs of light minimum.

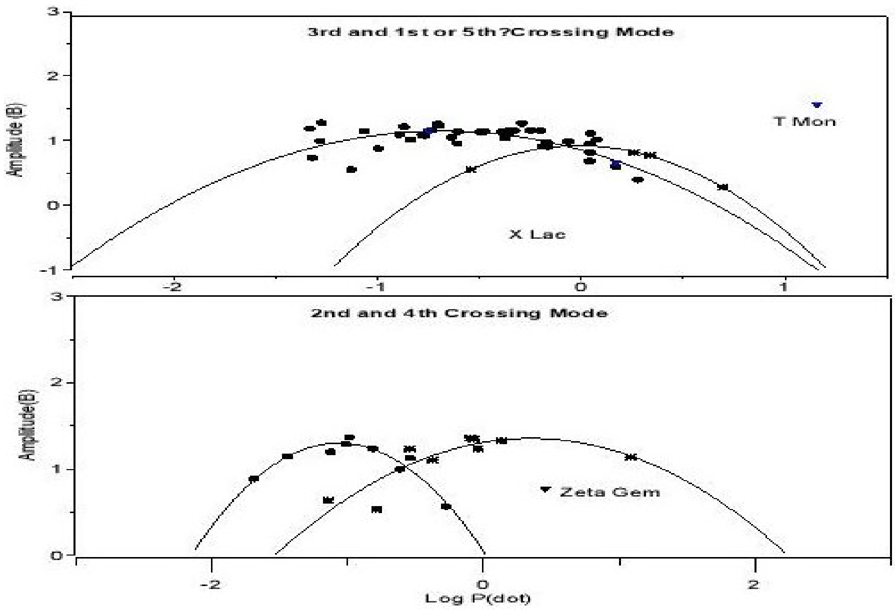

Fig. 4 – Normalized blue amplitude of Cepheids increasing pulsation periods as a function of normalized rate of period change. The upper section is the positive period change while the lower section is the negative period change.

As an illustration of the usefulness of Cepheid period changes as a diagnostic tool, we plotted in Fig. 4 the light curve amplitude A_B as a function of rate of period change *dP/dt* for Cepheid's (from Abdel-Sabour (2007)) and the sample of Cepheid's under focus for this study. The rate of period change *dP/dt* depends upon where a Cepheid is located in the instability strip, those with the fastest rates of period change must be located on the hot edge of the instability strip (since they correspond to higher mass stars), while those with the slowest rates of period change must be located on the cool edge of the instability strip (since they correspond to lower mass stars). As illustrated in Fig 4, pulsation amplitude, as measured by light curve,

Table 4

The least square fit straight line for cycle-to-cycle fluctuation

Star Name	$2a^2$	e
U Aql	-0.0105±0.0533	0.000±(0.0011)
η Aql	-0.1763±0.0179	0.0029±(0.0001)
ζ Gem	0.0492±0.0354	0.0001±(0.0001)
X Lac	0.0226±0.0691	0.0001±(0.0003)
T Mon	0.2584±0.0281	0.0004±(0.0001)
T Vul	0.0118±0.0908	0.00001±(0.00032)

enabled us to distinguish possible crossing modes for individual Cepheids. The dependence of pulsation amplitude on location in the instability strip was previously difficult to establish from observational parameters but it is simple to distinguish by using the rate of period change.

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