WAVE PROPERTIES OF ISOTHERMAL PLASMA AROUND SCHWARZSCHILD ANTI-DE SITTER BLACK HOLE

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Abstract. This paper investigates wave properties of isothermal plasma around the Schwarzschild Anti-de Sitter (SAdS) black hole. For this black hole, the 3+1 GRMHD equations are re-formulated which are linearly perturbed and then Fourier analyzed for rotating magnetized plasmas. The graphs of the wave number, the phase and group velocities with the refractive index are used to discuss the wave properties.

Key words: isothermal plasma – (3+1) formalism – Rindler coordinates – GRMHD – SAdS spacetime.

1. INTRODUCTION

One of the most intriguing predictions of general relativity is the existence of black holes. In fact, they belong to the most fascinating objects predicted by Einstein's field equations. Although many scientists have been studied from past several decades to prove conclusively the existence of black holes in the universe, black holes are still mysterious (Vachaspati *et al.*, 2007). Physicists are grappling the theory of black holes, while astronomers are searching for real-life examples of black holes in the universe (Narayan, 2005). There still exists no convincing observational data which can prove conclusively the existence of black holes in the universe. In recent years there has been renewed interest in investigating plasmas in the black hole environment. A successful study of the waves and emissions from plasmas falling into a black hole will be of great value in aiding the observational identification of black hole candidates. For this reason, plasma physics in the vicinity of a black hole has become a subject of great interest in astrophysics. In the immediate neighborhood of a black hole general relativity applies. It is therefore of interest to formulate plasma physics problems in the context of general relativity.

In order to investigate the behavior of plasma waves in the vicinity of a black hole, it would seem, in the first instance, to demand a covariant formulation based on the fluid equations of general relativity and Maxwell's equations in curved spacetime. But this approach has so far proved un-productive because of the curvature of four-dimensional spacetime in the region surrounding a black hole. Recently, Thorne, Price, and Macdonald (TPM) (Thorne *et al.*, 1982; Macdonald *et al.*, 1982; Price *et al.*, 1982; Thorne *et al.*, 1986) developed a method of a $3+1$ formulation of general relativity in which the electromagnetic equations and the plasma physics at least look somewhat similar to the usual formulations in flat spacetime while taking accurate account of general relativistic effects such as curvature. The 3+1 spacetime split was originally developed by Arnowitt, Deser, and Misner (ADM) (Arnowitt *et al.*, 1962) to study the quantization of the gravitational field. Since then, their formulation has most been applied in studying numerical relativity Evans *et al.*, 1986). TPM extended the ADM formalism to include electromagnetism and applied it to study electromagnetic effects near the Kerr black hole.

Sharif and Sheikh (2007) investigated the behavior of cold plasma waves in the vicinity of the Schwarzschild black hole horizon. The aim of this letter is to study the dynamical magnetosphere of the Schwarzschild Anti-de Sitter (SAdS) space using TPM formalism of the GRMHD equations, and investigate the nature of the waves. The motivation behind this study is based on some mentionable aspects. The solutions of black holes in Anti-de Sitter spaces are available from the Einstein equations with a negative cosmological constant. There exist a lot of differences between de Sitter black holes and Anti-de Sitter black holes. The difference consisting in them is due to minimum temperatures that occur when their sizes are of the order of the characteristic radius of the Anti-de Sitter space. For larger Anti-de Sitter black holes, their red-shifted temperatures measured at infinity are greater. This implies that such black holes can be in stable equilibrium with thermal radiation at a certain temperature. Moreover, recent development in string M-theory greatly stimulate the study of black holes in anti-de Sitter spaces. One example is the AdS/CFT correspondence (Hawking *et al.*, 1999; Chamblin *et al.*, 1999; Mann *et al.*, 1999) between a weakly coupled gravity system in an Anti-de Sitter background and a strongly coupled conformal field theory on its boundary. So the study on the Schwarzschild Anti-de Sitter black holes is pragmatic and having an important effect.

At first we summarize the GRMHD equations in the SAdS black hole magnetosphere in 3+1 formalism and then we investigate the GRMHD equations for isothermal plasma in the case of rotating magnetized surroundings. Finally, we present our remarks. We use units $G = c = 1$.

2. THE 3+1 SPLIT OF THE SADS BLACK HOLE SPACETIME

In this section, applying TPM formulation, we split the spacetime of the Schwarzschild-Anti-de Sitter black hole, which is the solution of Einstein equations with a negative $\Lambda(= 3/\ell^2)$ term corresponding to a vacuum state spherically symmetric configuration. The metric of the spacetime has the form

$$\begin{aligned} ds^2 &= g_{\mu\nu}dx^\mu dx^\nu \\ &= -\Delta^2 dt^2 + \frac{1}{\Delta^2}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \end{aligned} \tag{1}$$

where

$$\Delta^2 = 1 - \frac{2M}{r} - \frac{r^2}{\ell^2}. \tag{2}$$

Here, M is the mass of the black hole and the coordinates are such that $-\infty < t < \infty$, $r \geq 0$, $0 \leq \theta \leq \pi$ and $0 \leq \phi \leq 2\pi$. The metric (1) at large r represents the asymptotically Anti-de Sitter spacetime.

The SAdS black hole's horizon is situated at root of the cubic equation

$$r^3 - \ell^2 r + 2M\ell^2 = 0. \tag{3}$$

The only real root of this equation is

$$r_h = \frac{2}{3}\sqrt{3}\ell \sinh\left[\frac{1}{3}\sinh^{-1}\left(3\sqrt{3}\frac{M}{\ell}\right)\right]. \tag{4}$$

Expanding r_h in terms of M with $\frac{1}{\ell^2} \ll \frac{M^2}{9}$, we obtain

$$r_h \approx 2M\left(1 - \frac{4M^2}{\ell^2} + \cdots\right). \tag{5}$$

The event horizon of the SAdS black hole is smaller than the Schwarzschild event horizon, $r_{\text{Sch}} = 2M$. Therefore, we can write $r_h = 2M\eta$ with $\eta < 1$.

An absolute three-dimensional space defined by the hypersurfaces of constant universal time t is described by the metric

$$ds^2 = g_{ij}dx^i dx^j = \frac{1}{\Delta^2}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{6}$$

The indices i, j range over $1, 2, 3$ and refer to coordinates in absolute space. The fiducial observers (FIDOs), *i.e.*the observers at rest with respect to this absolute space, measure their proper time τ using clocks that they carry with them and make local measurements of physical quantities. The FIDOs use a local Cartesian coordinate system with unit basis vectors tangent to the coordinate lines, given by

$$\mathbf{e}_{\hat{r}} = \Delta\frac{\partial}{\partial r}, \qquad \mathbf{e}_{\hat{\theta}} = \frac{1}{r}\frac{\partial}{\partial\theta}, \qquad \mathbf{e}_{\hat{\varphi}} = \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}. \tag{7}$$

For a spacetime viewpoint rather than a $3+1$ split of spacetime, the set of orthonormal vectors also includes the basis vector for the time coordinate:

$$\mathbf{e}_{\hat{0}} = \frac{d}{d\tau} = \frac{1}{\alpha}\frac{\partial}{\partial t}. \tag{8}$$

Here α is the lapse function (or redshift factor) defined by

$$\alpha(r) \equiv \frac{d\tau}{dt} = \left(1 - \frac{2M}{r} - \frac{r^2}{\ell^2}\right)^{\frac{1}{2}}. \tag{9}$$

The gravitational acceleration felt by a FIDO is then correspondingly given by

$$\mathbf{a} = \nabla \ln\alpha = \frac{1}{\alpha}\left(\frac{M}{r^2} - \frac{r}{\ell^2}\right)\mathbf{e}_{\hat{r}}, \tag{10}$$

while the rate of change of any scalar physical quantity or any three-dimensional vector or tensor, as measured by a FIDO, is given by the derivative

$$\frac{D}{D\tau} \equiv \left(\frac{1}{\alpha}\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla\right), \tag{11}$$

$\mathbf{V}$ being the velocity of a fluid as measured locally by a FIDO.

The first metric in (6) is approximated in Rindler coordinates by

$$ds^2 = -\alpha^2 dt^2 + dx^2 + dy^2 + dz^2, \tag{12}$$

where

$$x = r_h\left(\theta - \frac{\pi}{2}\right), \qquad y = r_h\varphi, \qquad z = 2r_h\Delta. \tag{13}$$

The standard lapse function α takes the form $z/(2r_h)$ in Rindler coordinates. This function vanishes at the horizon which we can place at $z = 0$ and it increases monotonically as z increases from 0 to ∞.

Using 3+1 split of spacetime Maxwell's equations take the following form:

$$\nabla\cdot\mathbf{B} = 0, \tag{14}$$

$$\nabla\cdot\mathbf{E} = 4\pi\rho_e, \tag{15}$$

$$\frac{\partial\mathbf{B}}{\partial t} = -\nabla\times(\alpha\mathbf{E}), \tag{16}$$

$$\frac{\partial\mathbf{E}}{\partial t} = \nabla\times(\alpha\mathbf{B}) - 4\pi\alpha\mathbf{j}, \tag{17}$$

where ρ_e and $\mathbf{j}$ are electric charge and current density, respectively. For the perfect MHD (*i.e.*MHD with perfectly conducting) assumption there exists no electric field in the fluid's rest frame, *i.e.*$\mathbf{E} + \mathbf{V}\times\mathbf{B} = 0$.

Under this condition the equation for the evolution of magnetic field (16) becomes

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times(\alpha\mathbf{V}\times\mathbf{B}) = (\mathbf{B}\cdot\nabla)(\alpha\mathbf{V}) - \mathbf{B}\nabla\cdot(\alpha\mathbf{V}) - (\alpha\mathbf{V}\cdot\nabla)\mathbf{B}. \tag{18}$$

The conservation of mass, energy and momentum equations are written, respectively, as follows:

$$\begin{aligned}&\frac{\partial(\rho_o\mu)}{\partial t}+\{(\alpha\mathbf{V})\cdot\nabla\}(\rho_o\mu)+\rho_o\mu\gamma^2\mathbf{V}\cdot\frac{\partial\mathbf{V}}{\partial t}+\\&+\rho_o\mu\gamma^2\mathbf{V}\cdot(\alpha\mathbf{V}\cdot\nabla)\mathbf{V}+\rho_o\mu\{\nabla\cdot(\alpha\mathbf{V})\}=0,\end{aligned}\tag{19}$$

$$\begin{aligned}&\left\{\left(\rho_o\mu\gamma^2+\frac{\mathbf{B}^2}{4\pi}\right)\delta_{ij}+\rho_o\mu\gamma^4V_iV_j-\frac{1}{4\pi}B_iB_j\right\}\frac{DV^j}{D\tau}+\rho_o\mu\gamma^2V_i\frac{D\mu}{D\tau}-\\&-\left(\frac{\mathbf{B}^2}{4\pi}\delta_{ij}-\frac{1}{4\pi}B_iB_j\right)V^j{}_{,k}V^k=-\rho_o\mu\gamma^2a_i-p_{,i}+\\&+\frac{1}{4\pi}(\mathbf{V}\times\mathbf{B})_i\nabla\cdot(\mathbf{V}\times\mathbf{B})-\frac{1}{8\pi\alpha^2}(\alpha\mathbf{B})^2_{,i}+\frac{1}{4\pi\alpha}(\alpha B_i)_{,j}B^j-\\&-\frac{1}{4\pi\alpha}[\mathbf{B}\times\{\mathbf{v}\times(\nabla\times(\alpha\mathbf{v}\times\mathbf{B}))\}]_i,\end{aligned}\tag{20}$$

$$\begin{aligned}&\gamma^2\frac{D(\mu\rho_0)}{D\tau}-\frac{1}{\alpha}\frac{\partial p}{\partial t}+2\rho_0\mu\gamma^4\mathbf{V}\cdot\frac{D\mathbf{V}}{D\tau}+2\rho_0\mu\gamma^2(\mathbf{V}\cdot\mathbf{a})+\rho_0\mu\gamma^2(\nabla\cdot\mathbf{V})+\\&+\frac{1}{4\pi\alpha}\left[(\mathbf{V}\times\mathbf{B})\cdot(\nabla\times(\alpha\mathbf{B}))+(\mathbf{V}\times\mathbf{B})\cdot\frac{\partial}{\partial t}(\mathbf{V}\times\mathbf{B})\right]=0.\end{aligned}\tag{21}$$

Here a subscript i on a vector quantity refers to the i component of that vector. Equation (21) is derived by using

$$\epsilon=\{\mu\rho_0-p(1-\mathbf{V}^2)\}\gamma^2,\qquad \mathbf{S}=\mu\rho_0\gamma^2\mathbf{V},\qquad \overleftrightarrow{\mathbf{W}}=\mu\rho_\mathbf{0}\gamma^\mathbf{2}\mathbf{V}\otimes\mathbf{V}+p\overleftrightarrow{\gamma}\tag{22}$$

in equation (23)

$$\frac{d\epsilon}{d\tau}+\theta\epsilon+\frac{1}{2\alpha}W^{ij}(\pounds_t\gamma_{ij})=-\frac{1}{\alpha^2}\nabla\cdot(\alpha^2\mathbf{S})+\frac{1}{\alpha}(\nabla\boldsymbol{\beta}):\overleftrightarrow{\mathbf{W}}+\mathbf{E}\cdot\mathbf{j}.\tag{23}$$

Here ϵ, $\mathbf{S}$, $\overleftrightarrow{\mathbf{W}}$, $\overleftrightarrow{\gamma}$, θ, $\boldsymbol{\beta}$, $\otimes$ and $\pounds_t$ represent the mass energy density, energy flux, stress tensor, the three metric in absolute space, the expansion rate of the FIDO's four-velocity, the shift vector, the tensor product and the time derivative along shifting congruence (Lie derivative with respect to global time in a standard style). $\frac{d}{d\tau}\equiv\frac{1}{\alpha}\frac{\partial}{\partial t}$ is the rate of change of a three-dimensional vector which lies in the absolute space according to the FIDO. The $\mu\equiv(\rho+p)/\rho_o$ is the specific enthalpy of the fluid, where ρ is the total density of mass-energy and p is the pressure as seen in the fluid's rest frame. The ρ_o is the fluid's rest-mass density and $\gamma\equiv(1-\mathbf{V}^2)^{-1/2}$ is the fluid's Lorentz factor as seen by the FIDO's. Equations (18)-(21) are the perfect GRMHD equations for the SAdS black hole.

We consider for ease the "isothermal plasma"(consider the existence of pressure), for which the equation of state can be expressed as

$$\mu = \frac{\rho + p}{\rho_0} = \text{constant}. \tag{24}$$

Using (24) in (18)-(21) we get the perfect GRMHD for isothermal plasma close to the event horizon of SAdS black hole. We characterize the perturbed flow in the magnetosphere by its velocity $\mathbf{V}$ and magnetic field $\mathbf{B}$ as measured by the FIDO's, pressure of the fluid p and the fluid's density ρ. The first order perturbations in these quantities are denoted by $\delta\mathbf{V}$, $\delta\mathbf{B}$, δp and $\delta\rho$. Accordingly, the perturbed variables take the following form:

$$\begin{aligned} \mathbf{B} = \mathbf{B}^o + \delta\mathbf{B} = \mathbf{B}^o + B\mathbf{b}, \mathbf{V} = \mathbf{V}^o + \delta\mathbf{V} = \mathbf{V}^o + \mathbf{v}, \\ \rho = \rho^o + \delta\rho = \rho^o + \rho\tilde{\rho}, p = p^o + \delta p = p^o + p\tilde{p} \end{aligned} \tag{25}$$

where $\mathbf{B}^o$, $\mathbf{V}^o$, p^o and ρ^o are unperturbed quantities. The waves can propagate in z-direction due to gravitation with respect to time t and thus perturbed quantities must depend on z and t.

We use the linear perturbation and Fourier analyze techniques to reduce GRMHD equations to ordinary differential equations. The magnetosphere has the perturbed flow along x-z plane in this surroundings. The FIDO-measured fluid four-velocity can be described in this plane by $\mathbf{V} = V(z)\mathbf{e}_x + u(z)\mathbf{e}_z$, while the Lorentz factor is $\gamma = \frac{1}{\sqrt{1-u^2-V^2}}$. The rotating magnetic field can be expressed in the x-z plane as $\mathbf{B} = B[\lambda(z)\mathbf{e}_x + \mathbf{e}_z]$. The relation between the variables λ , u and V is $V = \frac{V_F}{\alpha} + \lambda u$, where V_F is an integration constant.

Using linear perturbation (25), we get a set of equations and then write the component form of these, and finally derive the following Fourier analyzed equations using

$$-c_3\{(\alpha\lambda)' + ik\alpha\lambda\} + c_4(\alpha' + ik\alpha) - c_6\{(\alpha u)' - i\omega + ik\alpha u\} = 0, \tag{26}$$

$$c_5(-\frac{i\omega}{\alpha} + iku) = 0, \tag{27}$$

$$ikc_5 = 0, \tag{28}$$

$$\begin{aligned} c_1\{\rho(-i\omega + iku\alpha) - \alpha' up - \alpha u' p - \alpha u p' - \alpha u p\gamma^2(uu' + VV')\} + \\ + c_2\{\rho(-i\omega + iku\alpha) + \alpha' up + \alpha u' p + \alpha u p' + \alpha u p\gamma^2(uu' + VV')\} + \\ + c_3(\rho + p)[-i\omega\gamma^2 u + ik\alpha(1 + \gamma^2 u^2) - \\ - \alpha\{(1 - 2\gamma^2 u^2)(1 + \gamma^2 u^2)\frac{u'}{u} - 2\gamma^4 u^2 VV'\}] \\ + c_4(\rho + p)\gamma^2[(-i\omega + ik\alpha u)V + \alpha u\{(1 + 2\gamma^2 V^2)V' + 2\gamma^2 uVu'\}] = 0, \end{aligned} \tag{29}$$

$$
\begin{aligned}
&c_1\rho\gamma^2u\{(1+\gamma^2V^2)V'+\gamma^2uVu'\}+c_2p\gamma^2u\{(1+\gamma^2V^2)V'+\gamma^2uVu'\}+\\
&+c_3[-\{(\rho+p)\gamma^4uV-\frac{\lambda B^2}{4\pi}\}\frac{i\omega}{\alpha}+\\
&+iku\{(\rho+p)\gamma^4uV+\frac{\lambda B^2}{4\pi}\}+(\rho+p)\gamma^2\{\{(1+2\gamma^2u^2)(1+2\gamma^2V^2)-\\
&-\gamma^2V^2\}V'+2\gamma^2(1+2\gamma^2u^2)uVu'\}+\frac{B^2u}{4\pi\alpha}(\alpha\lambda)']+\\
&+c_4[-\{(\rho+p)\gamma^2(1+\gamma^2V^2)+\frac{B^2}{4\pi}\}\frac{i\omega}{\alpha}+iku\{(\rho+p)\gamma^2(1+\gamma^2V^2)-\\
&-\frac{B^2}{4\pi}\}+(\rho+p)\gamma^4u\{(1+4\gamma^2V^2)uu'+4(1+\gamma^2V^2)VV'\}-\\
&-\frac{B^2u\alpha'}{4\pi\alpha}]-c_6\frac{B^2}{4\pi}\{ik(1-u^2)+(1-u^2)\frac{\alpha'}{\alpha}-uu'\}=0,
\end{aligned}
\tag{30}
$$

$$
\begin{aligned}
&c_1\gamma^2\rho[a_z+u\{(1+\gamma^2u^2)u'+\gamma^2VuV'\}]+\\
&+c_2[\gamma^2p\{a_z+u\{(1+\gamma^2u^2)u'+\gamma^2uVV'\}\}+ikp+p']+\\
&+c_3[-\frac{i\omega}{\alpha}\{(\rho+p)\gamma^2\times(1+\gamma^2u^2)+\frac{\lambda^2B^2}{4\pi}\}+\\
&+iku\{(\rho+p)\gamma^2(1+\gamma^2V^2)-\frac{\lambda^2B^2}{4\pi}\}+\\
&+\{(\rho+p)\gamma^2\{u'(1+\gamma^2u^2)(1+4\gamma^2u^2)+\\
&+2u\gamma^2\times\{(1+2\gamma^2u^2)VV'+a_z\}\}-(\alpha\lambda)'\frac{\lambda B^2u}{4\pi\alpha}\}]+\\
&+c_4[-\frac{i\omega}{\alpha}\{(\rho+p)\gamma^4uV-\frac{\lambda B^2}{4\pi}\}+\\
&+iku\{(\rho+p)\gamma^4uV+\frac{\lambda B^2}{4\pi}\}+\{\gamma^4(\rho+p)\times\{u^2V'(1+4\gamma^2V^2)+\\
&+2V\{a_z+uu'(1+2\gamma^2u^2)\}\}+\frac{\lambda B^2\alpha'u}{4\pi\alpha}\}]+c_6\frac{B^2}{4\pi}\{ik\lambda(1-u^2)+\\
&+\lambda(1-u^2)\frac{\alpha'}{\alpha}-\lambda uu'+\frac{(\alpha\lambda)'}{\alpha}\}=0,
\end{aligned}
\tag{31}
$$

$$
\begin{aligned}
&c_1\gamma^2[\rho(-\frac{i\omega}{\alpha}+iku)+2\rho u\{a_z+\gamma^2(uu'+VV')\}+\\
&+\rho u'+\rho' u]+c_2[p\{\frac{-i\omega}{\alpha}(\gamma^2-1)+ik\gamma^2 u\}+2\rho\gamma^2 u\{a_z+\gamma^2(uu'+VV')\}\\
&+p\gamma^2 u'+p'\gamma^2 u]+c_3[(\rho+p)\{-2\gamma^4 u\frac{i\omega}{\alpha}+\\
&+ik\gamma^2(1+2\gamma^2u^2)-\gamma^2\frac{u'}{u}+6\gamma^6u^2(uu'+VV')+\gamma^4(uu'+VV')+2\gamma^4uu'\\
&+\gamma^2 a_z(1+2\gamma^2u^2)\}+\frac{B^2}{4\pi\alpha}\{\lambda(\alpha\lambda)'-u(\alpha\lambda)'(u\lambda-V)-\\
&-i\lambda(u\lambda-V)(\omega+ku\alpha)\}]+c_4[2\gamma^4(\rho+p)\{V(-\frac{i\omega}{\alpha}+iku)+\\
&+\{3\gamma^2uV(uu'+VV')+uV'+uVa_z\}\}+\\
&+\frac{B^2}{4\pi\alpha}\{-(\alpha\lambda)'+\alpha' u(u\lambda-V)+i(\omega+ku\alpha)(u\lambda-V)\}]\\
&+\frac{B^2}{4\pi\alpha}c_6[u(\alpha\lambda)'+\{\alpha'-u(\alpha u)'+ik(1-u^2)\}(u\lambda-V)]=0. \quad (32)
\end{aligned}
$$

From (27) or (28) we obtain c_5 is zero which gives $b_z=0$. Equating the determinant of the coefficients of c_1, c_2, c_3, c_4 and c_6 of (26), (29)-(32) to zero, we get a complex dispersion relation of the form

$$
\begin{aligned}
&A_1(z,\omega)k^4+B_1(z,\omega)k^3+C_1(z,\omega)k^2+D_1(z,\omega)k+E_1(z,\omega)+\\
&+i\{A_2(z,\omega)k^5+B_2(z,\omega)k^4+C_2(z,\omega)k^3+\\
&+D_2(z,\omega)k^2+E_2(z,\omega)k+F_2(z,\omega)\}=0. \quad (33)
\end{aligned}
$$

The different types of modes of waves are investigated here for $B>0$ and the wave number is in arbitrary direction to **B**. We use the lapse function $\alpha=\frac{z}{2r_h}$ where $r_h\approx 2M(1-\frac{4M^2}{\ell^2}+\cdots)\simeq\eta\times 2.948\times 10^5$cm, $\eta<1$ for a black hole mass $M\sim 1M_\odot$. We assume $\rho=1$ and $B^2=8\pi$. From the mass conservation law in three-dimensions we get $u=\frac{1}{\sqrt{2+z^2}}$. For simplicity, we also assume that $u=V$. We get $\lambda=1-\frac{\sqrt{2+z^2}}{z}$ by taking $V_F=1$, which shows that the magnetic field diverges close to the horizon.

Using these values in (33) we get values for k, from which we evaluate the phase velocity $v_p\equiv\frac{\omega}{k}$ and group velocity $v_g\equiv(n+\omega\frac{dn}{d\omega})^{-1}$, where $n(=1/v_p)$ is the refractive index computed as the ratio of the speed of light in a vacuum to the speed of light through the material, and $\frac{dn}{d\omega}$ determines whether the dispersion is normal or not.

We get four real values of k from the real part of (33). Out of these two are real and interesting. The other two values are not interesting in the judgment that these turn out to be imaginary in the whole region. The imaginary part gives five values of k, out of which one is real but not interesting and others are complex conjugate. Here we elucidate only one dispersion relation obtained from the real part, is shown in the Figure 1.

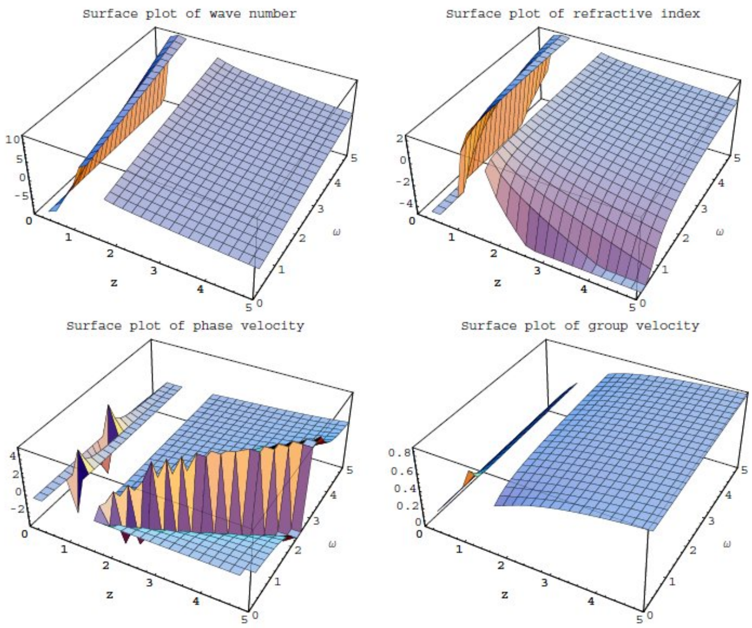

Fig. 1 – The region shows not normal dispersion, $v_g > v_p$ except some points, $\frac{dn}{d\omega} > 0$, but $n < 1$ (not shown in here).

We see in the Figure 1 that the wave number is huge large close to the event horizon and the waves lose energy as we go away from the event horizon of SAdS black hole. This shows that the increase in ω increases k and the waves are in growing mode as z decreases. The wave number takes negative values for some region. The group velocity is greater than the phase velocity except some points. Since $n < 1$ though $\frac{dn}{d\omega} > 0$, the region is not of normal dispersion.

3. DISCUSSION AND CONCLUDING REMARKS

In summary, we get that the wave number turns out to be infinite at the event horizon and consequently, no wave is present there due to immense gravitational field. This indicates that no signal can pass the event horizon or near to it. But when we depart from horizon, the waves lose energy. Therefore the waves are in damping mode as we go away from the horizon and in growing mode as we approach the horizon. It is known that the MHD waves in isothermal plasma are non-dispersive.

However, the dispersion is noted in the above figures. This factor comes due to the formalism used and the equations which provides different equations from the usual MHD equations.

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