ABOUT ORBITAL AND ROTATIONAL CHANGES OF (99942) APOPHIS DURING THE 2029 CLOSE ENCOUNTER WITH THE EARTH

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Abstract. NEA's constitute a potential threat of collisions with the Earth in the future and in consequence their study is fundamental. One of the most impressive phenomena related to these objects in the next decades is the close encounter of (99942) Apophis on April 13, 2029. This asteroid, with a diameter estimated at approximatively 300m, will approach the Earth at a distance of roughly 38 300 km from our planet. In this paper we present at first a detailed study of the orbital characteristics of Apophis and their changes during the 2029 encounter. Then we calculate the dramatic rotational effects undergone by the asteroid during the close encounter, depending on a few physical and geometrical parameters.

Key words: Near–Earth Asteroids, Apophis, orbital motion rotation..

1. INTRODUCTION

The asteroid (99942) Apophis was discovered by Tucker *et al.*at Kitt Peak Observatory on June 19, 2004. Observed during two consecutive nights, it was lost and re-observed six months later from Siding Spring, Australia on December 18,2004, by G. Garrad. Then calculations showed rapidly that Apophis could be a potential threat for our planet. These calculations were more and more refined with the increasing number of observational constraints. At the present time they show that during the 2029 close encounter, occuring on April 13th. the asteroid should approach our planet with a distance of roughly 38000 km, corresponding more or less to the geocentric distance of geostationary satellites. Some other close encounters are scheduled on 2068, 2085 and 2088, with risks of collision which can be considered as negligible.

In this paper we aim at developing two kinds of studies. First we want to give

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some insight on the orbital changes undergone by Apophis during the 2029 close encounter with our planet, quoted as the CE in the following. Then we want to extend simulations already partially undertaken by Souchay *et al.*(2014) concerning the variations of orientation of the rotational axis of the asteroid during the short time interval of time (a few hours) of the CE.

In Section 2 we give some information concerning orbital changes undergone by Apophis during the CE. In Section 3 we establish the theoretical foundations, taken from Kinoshita (1977) from which we calculate the variations in space of the axis of rotation or of figure, of the asteroid. Finally in Section 4, we show the various possibilities of variations of the axis in function of Apophis moments of inertia and two fundamental angular parameters λ_0 and ε_0, the last one representing the initial obliquity of the asteroid.

2. ORBITAL CHANGES OF APOPHIS DURING THE 2029 CLOSE ENCOUNTER

In order to determine the orbital changes undergone by the asteroid during the CE we start from the rectangular coordinates given by up-to-date ephemerides : one is the HORIZONS ephemerides from JPL, the other one can be found at the electronic portal of the IMCCE (Institut de Mécanique Céleste et de Calcul des Ephémérides) based at Paris observatory. They both give the rectangular coordinates of the position and velocity of the asteroid, with respect to the equinox and ecliptic of J2000.0. In order to convert the rectangular coordinates into osculating orbital elements we use classical formula related to keplerian motion, starting from position and velocity heliocentric vectors. Therefore we calculated the values of the six fundamental orbital parameters coming from the two ephemerides above. The differences between them are small enough to be ignored in the following.

According to the ephemerides above, the exact time of the minimum distance is JD 2462240.40694 which corresponds to April 13, 2029 at $21^h\ 46^{mn}$ UTC. This minimum distance is $d_{AE} = 2.5611653610^{-4}\ AU$, that is to say roughly 38 314 km.

In Table 1 we present the orbital characteristics of Apophis before and after the CE. Notice that before the CE Apophis semi-major axis satisfies the conditions $a < 1A.U.$ and $Q > 0.983A.U.$. These conditions range it inside the Aten's class of NEAs. On the other side, after the CE $a > 1A.U.$ and $q < 1.017A.U.$ which ranges the asteroid in the Apollo's class.

2.1. ORBITAL PARAMETERS VARIATIONS DURING THE CLOSE ENCOUNTER

In the small time interval ($\approx$ 1 day) of the CE, the orbital status of Apophis changes completely. The nature of its orbit changes drastically from a quasi-heliocentric to a quasi-geocentric one. Therefore, we can distinguish several steps. First, as far

Table 1

Orbital parameters of the asteroid (99942) Apophis before and after the 2029 close encounter with the Earth

Parameter	Before	After
Eccentricity (e)	0.191116567578	0.188586010733
Semi-major axis (a) [AU]	0.922432239776	1.101692916046
Inclination (i) [$^\circ$]	3.341355553915	2.234257393927
Distance to perihely : $q = a(1-e)$ [AU]	0.746140156287	0.893929043955
Distance to aphely : $Q = a(1+e)$ [AU]	1.098724323265	1.309456788136
Longitude of the asending node : Ω [$^\circ$]	204.618926447	204.293380282
Argument of perihely : ω [$^\circ$]	126.5495460	71.88936989
Revolution period : T [day]	323.5569	422.3182

as the asteroid distance to the Earth is large enough (step 1), Apophis orbital motion corresponds to a classical two-body problem perturbed by the planets, and we can assimilate its orbit at first approximation as an osculating keplerian heliocentric elliptic one. Second, when the Earth's attraction becomes significant with respect to the Sun (step 2), its orbital motion corresponds at first approximation to a restricted three-body problem, the mass of the asteroid being completely negligible with respect to that of the two perturbing bodies (Earth and Sun). During the small interval of time of roughly a few hours before and after the CE (step 3), the gravitational influence of the Earth is by far dominant with respect to that of the Sun. In that case the motion can be considered as an hyperbolic motion slightly perturbed by the Sun and the Moon. Then we come back successively to the two first configurations (steps 4 and 5) symmetrical respectively to steps 2 and 1. The purpose here is not to recompute the ephemerides of Apophis during the whole CE, but to give explicitly some information concerning its orbital characteristics, as well as to answer to some basic questions : what is the ratio between the Newton's force exerted by the Earth to that exerted by the Sun ? Is the gravitational effect of the Moon large enough to perturb significantly the orbit ? What are the changes of the orbital parameters between step 1 and step 5 ? What are the orbital parameters characterizing the hyperbolic motion of step 3 ? etc.

In Fig.1 we show the relative distance of the Earth and of the Moon from the asteroid, with respect to the time. The first one (Apophis-Earth) falls down from 0.0073 AU to the minimum value of $d_{AE} = 0.000256116536$ AU $\approx 38\,314$ km in an interval of time of 50 hours, whereas the second one (Apophis-Moon) reaches a minimum $16^{\mathrm{h}}50^{\mathrm{mn}}$ after the first one. This minimum is $d_{AM} = 6.365229 \times 10^{-4}$ AU $\approx 95\,222$ km which gives the ratio : $d_{AM}/d_{AE} = 2.49$. Notice that the mass of the Moon is roughly 1/81 the mass of the Earth and that $d_{AE} \ll d_{AM}$. Thus the global perturbing effect exerted by the Moon should considerably smaller than that exerted by the Earth.

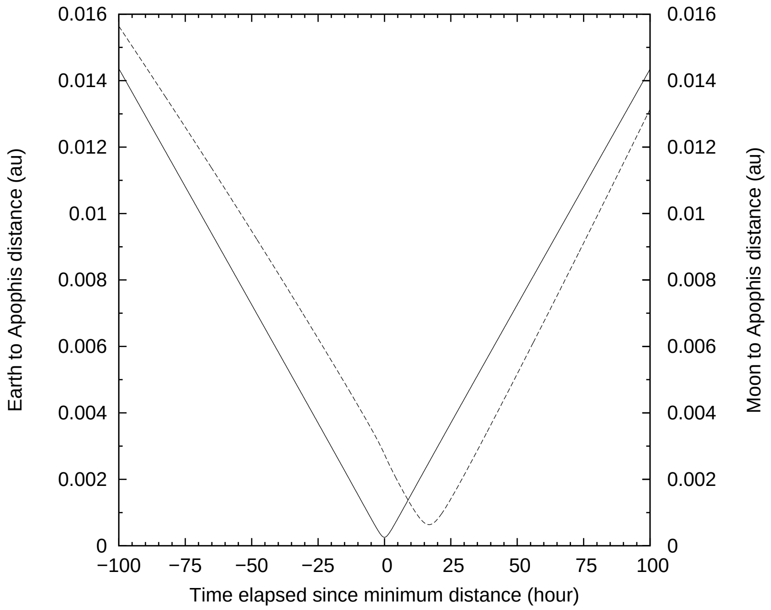

Fig. 1 – Relative distance Apophis-Earth (full) and Apophis-Moon (dashed) during the close encounter with the Earth. $t = 0$ corresponds to 2029, April 13, $21^{\mathrm{h}}\ 46^{\mathrm{mn}}$ UTC

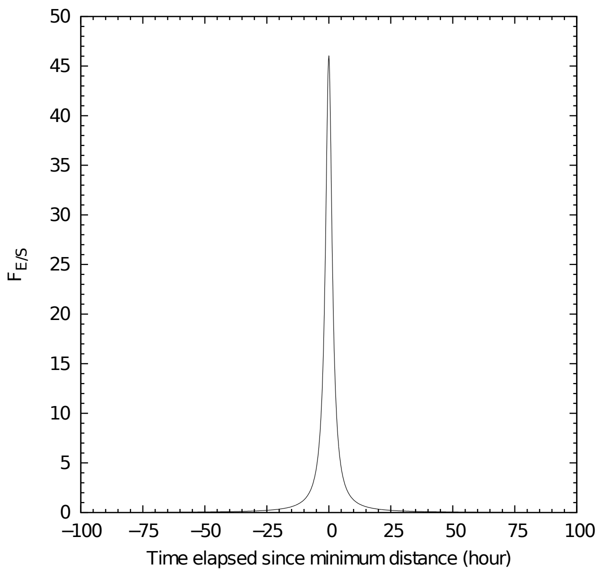

Fig. 2 – Ratio of the gravitational force exerted by the Earth , with respect to that exerted by the Sun, on Apophis. $t = 0$ corresponds to 2029, April 13, $21^{\mathrm{h}}\ 46^{\mathrm{mn}}$ UTC

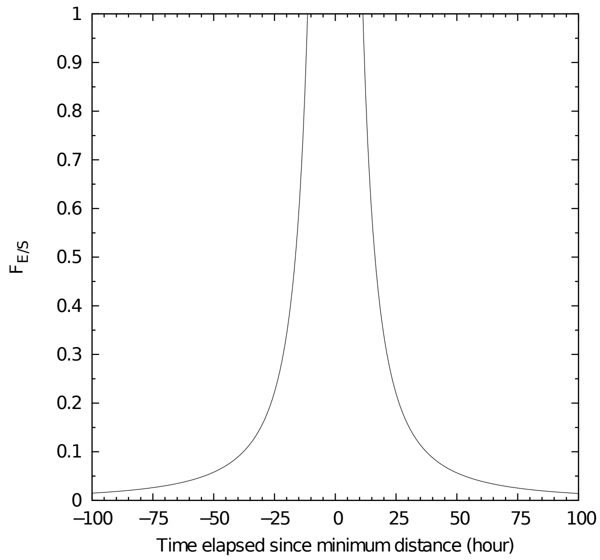

Fig. 3 – Ratio of the gravitational force exerted by the Earth , with respect to that exerted by the Sun, on Apophis. $t = 0$ corresponds to 2029, April 13, 21^{h} 46^{mn} UTC

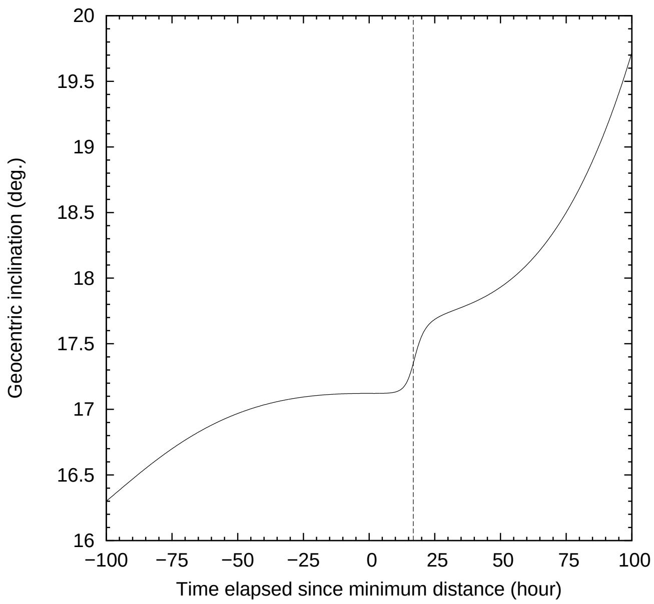

Fig. 4 – Variations of the geocentric inclination i_g of Apophis with respect to the ecliptic, during the CE. $t = 0$ corresponds to 2029, April 13, 21^{h} 46^{mn} UTC

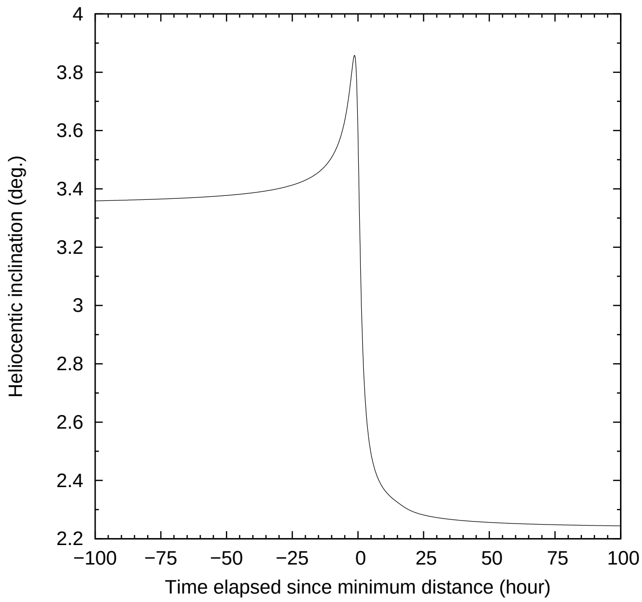

Fig. 5 – Variations of the heliocentric inclination i_h of Apophis with respect to the ecliptic, during the CE. $t = 0$ corresponds to 2029, April 13, $21^{\mathrm{h}}\ 46^{\mathrm{mn}}$ UTC

In Fig.2 we show the ratio ρ_{ES} of the Earth's gravitational attraction force with respect to that of the Sun. We observe that during the minimum distance this ratio is $\rho_{ES} \approx 45$. Moreover the peak is very sharp, and the two forces are equivalent for $t = \pm 15$h. In Fig.3 we zoom the zone for which $\rho_{ES} \ll 1$. The curve is quite symmetrical and we observe that $\rho_{ES} = 0.1$ for $t = \pm 35$h

It is worth determining the variations of inclination of Apophis' orbit due to the close encounter with respect to the ecliptic plane of J2000.0 both in a geocentric and an heliocentric reference frame. In Fig.4 we show the inclination i_g with respect to the geocentric frame : results are quite consistent with what could be expected. From $t = -15$h to $t = +15$h the curve is flat: the reason is that we are in step 3, in the case of a two-body problem Apophis-Earth not strongly perturbed by the Sun and the Moon. Therefore it is quite logical that the inclination remains quasi-constant. On the other hand, outside this interval the inclination is varying significantly with a positive slope, between $i_g = 16.3°$ to $i_g = 19.7°$ in an interval of time of 200h. Nevertheless we can point out a very important dissymmetry of the curve, due to the presence of the Moon, for which the instant ($t = 17$h) of minimum distance from the asteroid is represented by the dashed vertical line. The close encounter with our satellite creates a very strong relative increase of the slope of i_g. The consequence is an important difference between the increase of i_g during the 100 h preceding the

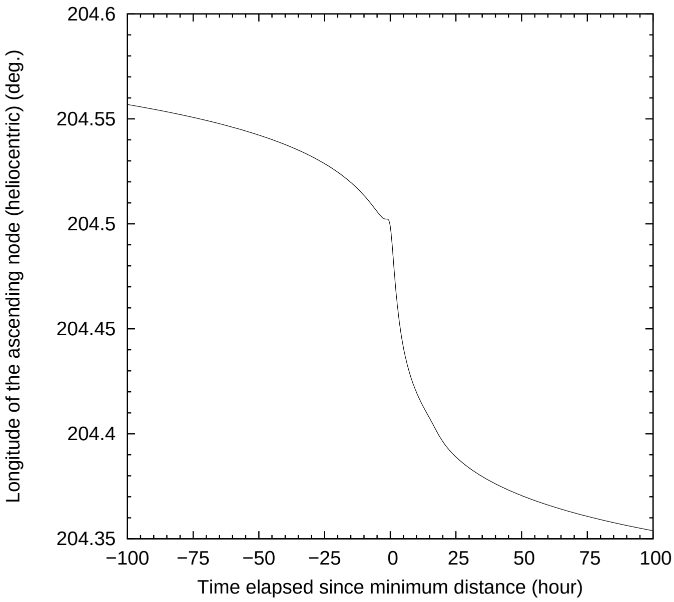

Fig. 6 – Variations of the longitude of the node Ω of Apophis with respect to the ecliptic, during the CE. $t = 0$ corresponds to 2029, April 13, 21^h 46^{mn} UTC

minimum distance ($\Delta i_g = 0.8°$) and the same increase in the 100h interval after the minimum distance ($\Delta i_g = 2.6°$).

The inclination i_h of Apophis orbit in a heliocentric reference frame, shown in Fig.5, presents a completely different aspect, still in accordance with the expectations. Outside the interval $t = [-100\text{h}, +100\text{h}]$, we are in the case of a two-body problem slightly perturbed by the Earth, the Moon and the other planets (steps 1 and 5). Therefore the values of i_h are quasi constant. On the other hand, the influence of the close encounter with the Earth- Moon system is a gradual exponential increase of i_h between $i_h = 3.37°$ to $i_h = 3.85°$ until $t = 0$, then a dramatically sharp decrease to $i_h = 2.40°$ at $= 10h$ and a final value of $i_h = 2.25°$ at $t = 100h$. For information we also present in Fig.6 the variation of longitude of the ascending node of Apophis orbit with respect to the ecliptic. We can remark that this variation is relatively small in comparison of i_h.

2.2. ORBITAL PARAMETRIZATION FOR ROTATIONAL MOTION

In the next section we will remark that the calculations of the perturbing rotational potential exerted by the Earth on Apophis necessitates the adoption of a conventional inertial reference plane. As already explained in similar rotational studies, for instance Souchay *et al.* (2003) in the specific case of the the asteroid Eros

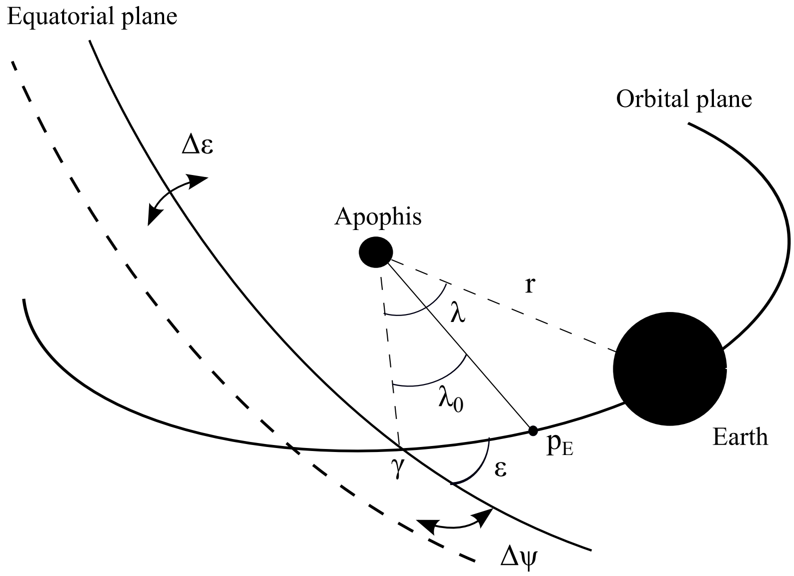

Fig. 7 – Parametrization of the Apophis-Earth system during the CE.

and Lhotka *et al.* (2013) in the case of a set of 100 asteroids, it is recommended to take this reference plane coinciding with the orbital plane of the asteroid around the perturbing body (the Earth), or by equivalence, the plane of the relative motion of the perturbing body with respect to the asteroid center of mass. We have already remarked that the orbital osculating orbital plane of Apophis with respect to the Earth (or in a reciprocal way of the Earth with respect to Apophis) during an interval of time $t = [-25\mathrm{h}, +10\mathrm{h}]$ can be considered as fixed with an inclination $i_g = 17.1^\circ$ with respect to the ecliptic. Therefore for the reasons explained above, we can choose this plane as the basic inertial reference one. We name it (P_0).

Then with this reference plane two parameters are enough to represent the relative motion of our planet with respect to the center of mass of the asteroid (Fig.7): these are r, the distance between the centers of mass, and $\tilde{\lambda}$ which is the longitude angle of the Earth center of mass with respect to a fixed direction which is taken as coincident with the direction of the Earth (namely $\mathrm{P_E}$) at its minimum distance ($\tilde{\lambda} = 0$ at this minimum). Now we define γ (by similitude with the vernal equinox) as the ascending node of the Earth orbital plane (with respect to Apophis) and λ_0 as the angle between γ and $\mathrm{P_E}$. To calculate the Earth disturbing potential in the following section we use as fundamental parameter the longitude of the Earth λ as calculated from γ, that is to say : $\lambda = \tilde{\lambda} + \lambda_0$

3. DISTURBING POTENTIAL DUE TO THE EARTH AND EQUATIONS OF MOTION

The theoretical frame to calculate the rotational variations of Apophis during the CE can be found in Kinoshita (1977). It is based on Hamiltonian theory involving Andoyer's canonical variables. It has been applied with a big accuracy by Souchay *et al.* (1999) at the level of $1\mu\mathrm{as}$ for the precession-nutation of the Earth, with an agreement at the level of a few $\mu\mathrm{as}$ with numerical integration (Souchay, 1998). It has also been used for the precise calculations of the orbital motion of asteroids (Petit *et al.*, 2014) and of terrestrial planets as Mars (Bouquillon and Souchay, 1999) and Venus (Cottereau *et al.*, 2010, 2011) undergoing the gravitational effect due to the Sun. In the present case Apophis is the studied body and the Earth is the perturbing one. The origin of the reference frame for our computations is Apophis center of mass. The basic plane is (P_0). We start from the expression of the perturbing potential exerted by the Earth, divided in a first order and a second order components, respectively U_1 and U_2.

$$U = U_1 + U_2 \tag{1}$$

with :

$$U_1 = \frac{\kappa^2 M_\oplus}{r^3}\left[\frac{2C-A-B}{2}P_2(\sin\delta) + \frac{A-B}{4}P_2^2(\sin\delta)\cos 2\alpha\right] \tag{2}$$

$$U_2 = \sum_{n=3}^{\infty}\frac{\kappa^2 M M_\oplus a_\oplus^n}{r^{n+1}}\cdot\left[J_n P_n(\sin\delta) - \right.$$
$$\left. - \sum_{m=1}^{n} P_n^m(\sin\delta)\times(C_{nm}\cos m\alpha + S_{nm}\sin m\alpha)\right] \tag{3}$$

where M stands for the mass of Apophis, $M_\oplus$ for that of the Earth. α is the longitude of the perturbing body (the Earth) counted from a conventional zero meridian on the asteroid (in that sense it must not be confused with the classical right ascension) and δ represents its declination with respect to Apophis equatorial plane. The P_i^j represent the Legendre polynomials of order i and degree j. Here we will restrict to the first order part U_1 of the potential. The Legendre polynomial $P_2(\sin\delta)$ can be developed in the following form (Kinoshita, 1977) :

$$
\begin{aligned}
P_2(\sin\delta) = & \frac{1}{2}(3\cos^2 J - 1)\Big[\frac{1}{2}(3\cos^2 I - 1)P_2(\sin\beta) - \frac{1}{2}\sin 2I P_1^2 \sin(\beta)\times \\
& \times \sin(\lambda - h) - \frac{1}{4}\sin 2I P_2^2(\sin\beta)\cos(2\lambda - h)\Big] + \\
& + \sin 2J\Big[-\frac{3}{4}\sin 2I P_2 \sin(\beta)\, cos\, g - \sum_{\epsilon=\pm1}\frac{1}{4}(1+\epsilon\cos I)\times \\
& \times(-1+2\epsilon\cos I)P_1^2(\sin\beta)\sin(\lambda - h - \epsilon g) - \\
& - \sum_{\epsilon=\pm1}\frac{1}{8}\epsilon\sin I(1+\epsilon cos\, I)P_2^2\sin(\beta)\cos(2\lambda - 2h - \epsilon g)\Big] + \\
& + sin^2 J\Big[\frac{3}{4}\sin^2 I P_2(\sin\beta)\cos 2g + \frac{1}{4}\sum_{\epsilon=\pm1}\epsilon\sin I\times \\
& \times(1+\epsilon\cos I)P_1^2\sin(\beta)\sin(\lambda - h - 2\epsilon g) - \frac{1}{16}\times \\
& \times\sum_{\epsilon=\pm1}(1+\epsilon\cos I)^2 P_2^2(\sin\beta)\cos 2(\lambda - h - \epsilon g)\Big]
\end{aligned}
\tag{4}
$$

For the sake of conciseness we do not provide here the corresponding complete expression for $P_2^2(\sin\delta)$, which can be found in Kinoshita (1977).

In the equation above, r and λ have already been defined previously. β represents the latitude of the perturbing body with respect to (P_0). From the definition of the plane (P_0) itself, we have : $\beta \approx 0$. $I = -\varepsilon$ stands for the obliquity, *i.e.* the angle between Apophis equatorial plane and (P_0). $h = -\psi$ is the angle of precession-nutation in longitude, *i.e.* the slow motion of the node γ of Apophis equatorial plane with respect to (P_0). For a better understanding of these variables we can refer to Fig.7. Notice that in all the formula above I and h represent the bi-dimensional motion of the axis of angular momentum of Apophis (Kinoshita, 1977). In general we are more concerned by both the motion of the axis of figure and of rotation in space rather than this last one. In fact, as it is the case for the Earth and the other terrestrial planets, we can postulate that these three axes are very close one to the other and that we can neglect their angular difference. This means that $J \approx 0$. With the simplifying hypotheses above ($\beta \approx 0$ and $J \approx 0$) the perturbing potential at first order U_1 reduces to the very simple following expression :

$$U_1 = U_{1,a} + U_{1,b} \tag{5}$$

$U_{1,a}$ and $U_{1,b}$ depending directly respectively of the flattening and trixiality :

$$U_{1,a} = \frac{\kappa^2 M_\oplus}{Gr^3}\frac{2C-A-B}{2}\left(-\frac{1}{4}(3\cos^2 I - 1) - \frac{3}{4}\sin^2 I \cos 2(\lambda - h)\right) \tag{6}$$

$$\begin{aligned} U_{1,b} = \frac{\kappa^2 M_\oplus}{Gr^3}\frac{A-B}{4}\Bigg(&\frac{3}{2}\sin^2 I \cos 2(l+g) + \frac{3}{4}(1+\cos I)^2 \cos 2(\lambda - h - l - g) \\ &+ \frac{3}{4}(1-\cos I)^2 \cos 2(\lambda - h + l + g)\Bigg) \end{aligned} \tag{7}$$

Then we define the following constants K and K' :

$$K = \frac{3GM_\oplus}{a^3\omega}H_D \text{ and } K' = \frac{3GM_\oplus}{a^3\omega}H_T \tag{8}$$

with

$$H_D = \frac{2C-A-B}{2C}\ ,\ H_T = \frac{A-B}{4C} \text{ and } G = C\omega \tag{9}$$

H_D is the dynamical ellipticity of the asteroid, H_T a coefficient of triaxiality, G is the amplitude of the angular momentum, C the moment of inertia along the z axis, and $\omega = 2\pi/T$ is the frequency of the rotation.

Following Kinoshita's theory and using the same kind of developments as preliminary ones by Souchay *et al.*(2014) we find the bi-dimensional variations of Apophis axis in space :

$$\begin{aligned} \Delta\psi = -\Delta h = &\frac{K}{2}\int \cos I\left(\frac{a}{r}\right)^3\Big(1 - \cos 2(\lambda - h)\Big)\mathrm{d}t \\ &+ \frac{K'}{2}\int\left(\frac{a}{r}\right)^3\Big(2\cos I\cos 2(l+g) - (1+\cos I)\cos 2(\lambda - h - l - g) \\ &+ (1-\cos I)\cos 2(\lambda - h + l + g)\Big)\mathrm{d}t \end{aligned} \tag{10}$$

$$
\begin{aligned}
\Delta\varepsilon = -\Delta I = \frac{K}{2}\int \sin I\left(\frac{a}{r}\right)^3 \sin 2(\lambda-h)\mathrm{d}t - K'\int \cos I\left(\frac{a}{r}\right)^3 \sin I \sin 2(l+g)\mathrm{d}t \\
-\frac{K'}{2}\int \frac{1}{\sin I}\left(\frac{a}{r}\right)^3\Big[(1+\cos I)^2\sin 2(\lambda-h-l-g) \\
+(1-\cos I)^2 \sin 2(\lambda-h+l+g)\Big]\mathrm{d}t \\
+\frac{K'}{2}\int \frac{\cos I}{\sin I}\left(\frac{a}{r}\right)^3\Big[(1+\cos I)^2\sin 2(\lambda-h-l-g) \\
-(1-\cos I)^2 \sin 2(\lambda-h+l+g)\Big]\mathrm{d}t
\end{aligned}
\tag{11}
$$

Notice that in this simplified expression of the potential the combination of Andoyer angles $(l+g)$ corresponds to the angle of proper rotation Φ of the asteroid, so that we can write : $\Phi = l+g = \omega t + \Phi_0$, where Φ_0 is an integration constant. These angles appear only in the triaxial part of the potential $U_{1,b}$, depending on K' and involving H_T.

4. SIMULATIONS AND RESULTS

Following the analytical developments and formula above, and in particular the final equations (10) and (11) we can deduce directly the bi-dimensional variations $(\Delta\psi, \Delta\varepsilon)$ of Apophis axis in space, once we know fundamental physical and geometrical parameters. These are the moments of inertia A, B, and C of the asteroid, the initial value of the obliquity ε_0 and of the longitude λ_0. r and $\tilde{\lambda}$ are given as a function of time by the numerical ephemerides as HORIZONS. H_D and H_T are directly deduced from the values of the moments of inertia. λ is determined from the relation : $\lambda = \tilde{\lambda} + \lambda_0$. The advantage of our calculation method lies in the fact that it depends only of a few parameters : ε_0, λ_0, H_D and H_T. In fact the values of these parameters are not still known in the case of Apophis, but should be revealed following observational campaigns during favourable conditions in particular just before the CE. Nevertheless we can carry out simulations to estimate the variability of $\Delta\psi$ and $\Delta\varepsilon$ in function of these parameters.

As an example, we give respectively in Fig.8 and Fig.9 the results of our simulations for $\varepsilon_0 = \pi/4$ and for four different values of λ_0. We assumed here that : $H_D = 0.2$ and $H_T = 0$ which means that Apophis is considered as an elongated asteroid with an axi-symmetrical form. We can observe that the amplitudes of the variations can reach a few degrees, which is dramatically large when we consider that they occur in a time interval of a few hours, whereas such variations necessitate in general hundred or thousand years for planets or asteroids.

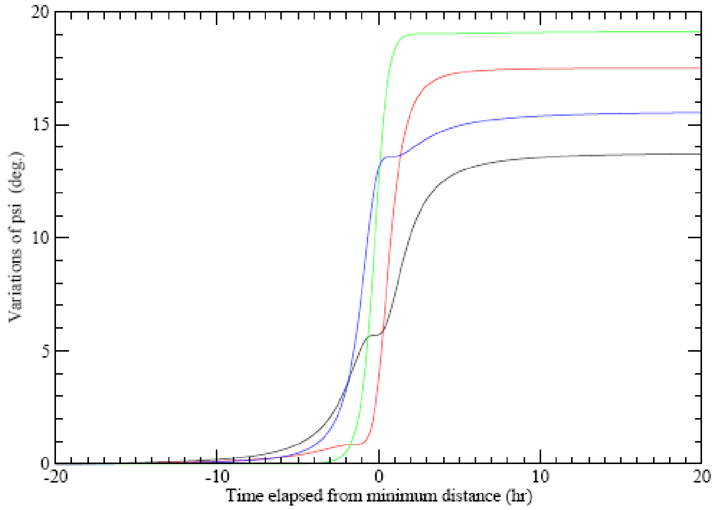

Fig. 8 – Variations of Apophis precession in longitude $\Delta\psi$ (cf. Fig.7) during the close encounter with the Earth, with $H_D = 0.2$,$H_T = 0.0$, $\varepsilon_0 = \pi/4$. The curves correspond to $\lambda_0 = 0$ (black), $\lambda_0 = \pi/4$ (red); $\lambda_0 = \pi/2$ (green); $\lambda_0 = 3\pi/4$ (blue).

In order to have an overall insight of the dependency of the amplitudes of the drift in $\Delta\psi$ and $\Delta\varepsilon$ with respect to initial conditions for ε_0 and λ_0 we plot 3-dimension diagrams respectively in Figs. 10 and Fig.11. We can remark that for $\Delta\psi$ (Fig.10) the drift is particularly important for values of λ_0 between $\pi/4$ and $3\pi/4$ and absolute values of ε_0 between $\pi/4$ and $\pi/2$. Its amplitude can reach 40 degrees for these intervals. For $\Delta\varepsilon$ (Fig.11), maximum amplitudes of the drift reach about ± 8 degrees for four zones displayed diagonally in the diagram, centered in $\lambda_0 = \pi/4$ and $\lambda_0 = 3\pi/4$.

Figs. 12 and 13 are the corresponding diagrams showing the peak-to-peak amplitudes of variations of $\Delta\psi$ and $\Delta\varepsilon$ instead of the final drifts. Therefore these amplitudes are naturally bigger, but in a rather small proportion. We can remark that the changes of obliquity (Fig.13) reach 10 degrees for an important percentage of cases.

5. CONCLUSION AND PROSPECTS

This study consists in a prolongation of a previous work carried out by Souchay *et al.*(2014). It generalizes this study by providing, for a given value of the dynamical

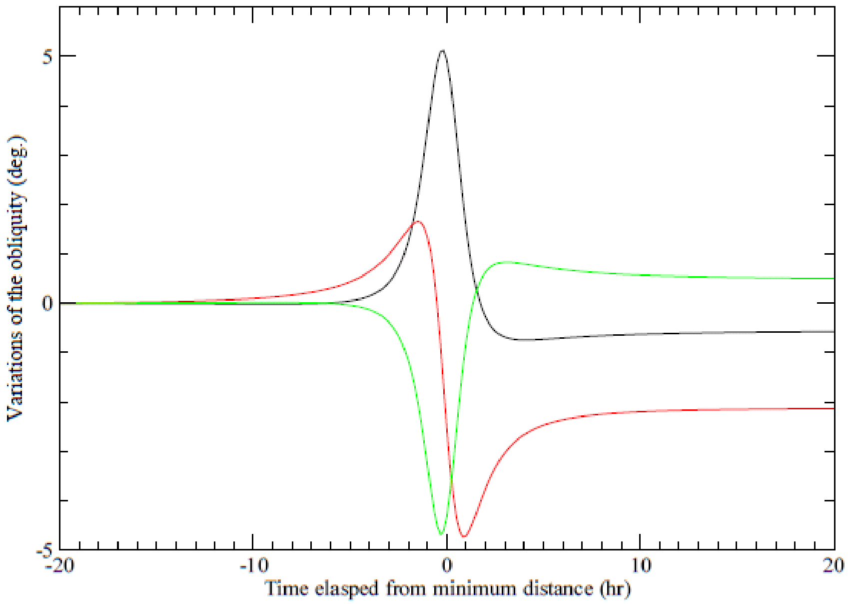

Fig. 9 – Variations of Apophis obliquity $\Delta\varepsilon$ (cf. Fig.7) during the close encounter with the Earth, with $H_D = 0.2$,$H_T = 0.0$, $\varepsilon_0 = \pi/4$. The curves correspond to $\lambda_0 = 0$ (black), $\lambda_0 = \pi/4$ (red); $\lambda_0 = \pi/2$ (green); $\lambda_0 = 3\pi/4$ (blue).

ellipticity H_D of (99942) Apophis, the overall structure of the drifts undergone by the obliquity and precession of the asteroid during its 2029 close encounter with the Earth, with respect to any value both of the initial obliquity ε_0 and of the position of the vernal equinox given by λ_0. In this paper we have assumed that the asteroid satisfies the condition of axi-symmetry which means that coefficient of triaxiality of the asteroid has been set to $H_T = 0$. An average value of the dynamical ellipticiy of the asetroids (Lhotka *et al.*, 2013) was chosen and fixed to $H_D = 0.2$.

We could prove that in most cases the asteroid will undergo dramatic changes of obliquity and precession, with amplitudes reaching potentially respectively 10 degrees and 40 degrees, which is considerable and could be easily detected by modern observational techniques as radar ones. The advantage of our model is that it can be adjusted as soon as the physical and geometrical knowledge of the asteroid will be refined, in particular in the years and weeks preceding the 2029 close encounter. Thus accurate of H_T, H_D, ε_0 and λ_0 will lead to a deterministic numerical calculation of the rotational changes of the asteroid available for comparisons with optical or radar observations of the CE.

Further studies are in preparation starting from a more complex form of the

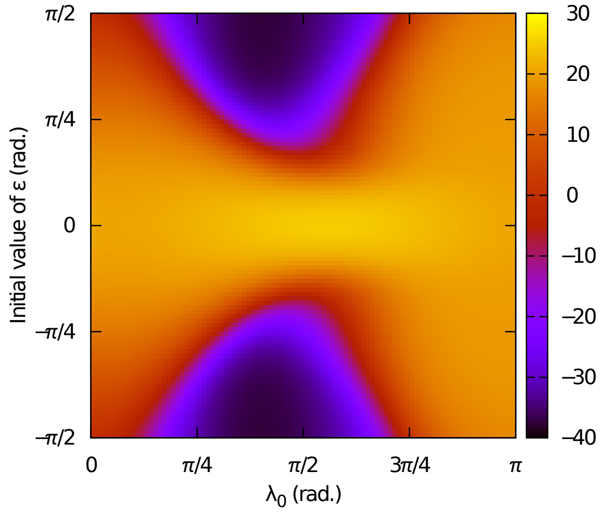

Fig. 10 – Three-dimensional diagram of the final difference in $\Delta\psi$ when comparing its value before and after the CE, as a function of λ_0 and ε_0.

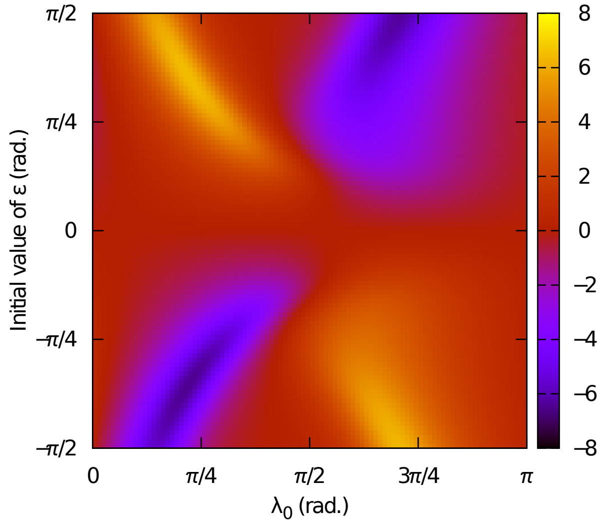

Fig. 11 – Three dimensional diagram of the final difference in $\Delta\varepsilon$ when comparing its value before and after the CE, as a function of λ_0 and ε_0.

asteroid with $H_T \neq 0$ and with the inclusion of second order parts of the potential exeterd by the Earth, given by (3). For this purpose we should use similar methods to Folgueira *et al.*(1998a,b). Moreover some non rigid effects should have to be taken into account as the effect of the tides exerted by the Earth, on H_D (Souchay and

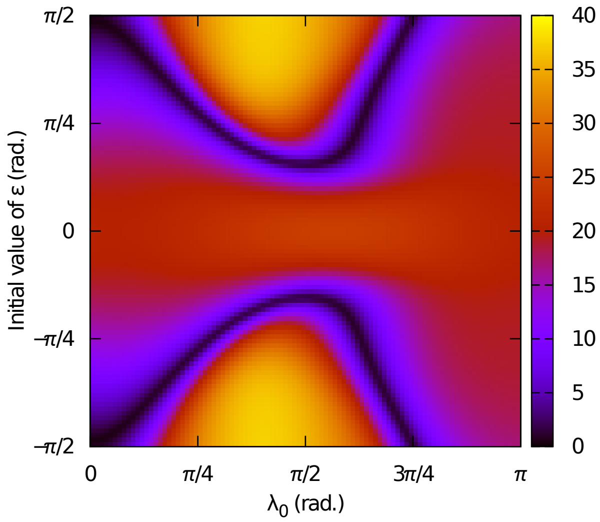

Fig. 12 – Three-dimensional diagram of the peak-to-peak amplitude in $\Delta\psi$ during the 20 hrs. before and after the CE, as a function of λ_0 and ε_0.

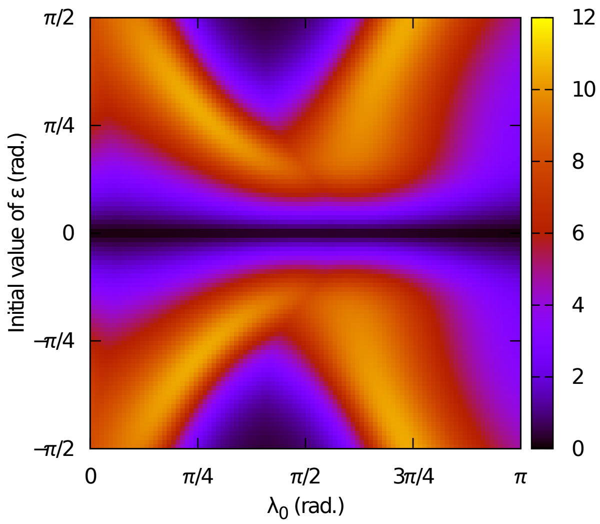

Fig. 13 – Three-dimensional diagram of the peak-to-peak amplitude in $\Delta\varepsilon$ during the 20 hrs. before and after the CE, as a function of λ_0 and ε_0.

Folgueira, 1998). At last the variations of the rotational angular velocity could be investigated and compared with Scheeres *et al.*(2005).

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